

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JULY 2013

(CCSS)

Mathematics

MM 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* twelve questions :

1 Evaluate $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

2 Find dy if $y = \sin 3x$.

3 Write the sums without sigma notation and then evaluate the sum $\sum_{k=1}^3 (-1)^{k+1} \sin \frac{\pi}{k}$.

4 Suppose that $\int_1^2 f(x) dx = 5$. Find $\int_1^2 -f(x) dx$.

5 Evaluate $\int_0^4 \left(3x - \frac{x^3}{4} \right) dx$.

6 Evaluate $\lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x + 5}$.

7 Define the continuity of a function f at a right end point $x = b$ of its domain.

8 Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$.

9 Find the slope of the curve $f(x) = x^2 + 1$ at $(2, 5)$.10 At what points do the graph of the function $f(x) = x^2 + 4x - 1$ has horizontal tangents.

11 State the mean value theorem.

12 The radius r of a circle increases from $r_0 = 10$ m to 10.1 m. Estimate the increase in the circles area A by calculating dA .(12 \times $\frac{1}{4}$ = 3 weightage)II. Short answer type questions. Answer all *nine* questions :

13 Find $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

Turn over

14 Find the absolute maximum and minimum value of $g(t) = 8t - t^4$ on $[-2, 1]$.

15 Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.

16 Find the linearization of $f(x) = x^3 - x$ at $x = 1$.

17 Evaluate $\sum_{k=1}^4 (k^2 - 3k)$.

18 Find the average value of $f(x) = -3x^2 - 1$ on $[0, 1]$.

19 Evaluate $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t \, dt$.

20 Find the volume of the solid generated by revolving the region bounded by :

$$y = x^2, y = 0, x = 2.$$

21 Where does the slope of the curve $y = \frac{1}{x}$ equal $-\frac{1}{4}$?

(9 × 1 = 9 weightage)

Short essay. Answer any five questions from seven :

22 Show that $\lim_{x \rightarrow 0^+} (1+x)^{1/x} = e$.

23 Prove that the function $y = |x|$ is differentiable on $(-\infty, 0)$ and $(0, \infty)$ but has no derivative at $x = 0$.

24 Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.

25 Express the solution of the following initial value problem as an integral

$$\text{Differential equation : } \frac{dy}{dx} = \tan x.$$

$$\text{Initial condition : } y(1) = 5.$$

26 Find the total area between the region $y = -x^2 - 2x$, $-3 \leq x \leq 2$ and the x -axis.

27 Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

- 28 Find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}$, $0 \leq x \leq 4$ about the x -axis.

(5 × 2 = 10 weightage)

Essay questions. Answer any *two* questions from three :

- 29 The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x -axis to generate a solid. Find the volume of the solid.
- 30 Find the area of the region in the first quadrant that is bounded and above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$.
- 31 Evaluate $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{e^\theta - \theta - 1}$.

(2 × 4 = 8 weightage)