

D 13017

## FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Maximum : 80 Marks

Time : Three Hours

## Section A

*Answer all questions.**Each question carries 1 mark.*

1. Find  $\text{Lt}_{t \rightarrow 1} \frac{t^2 + t - 2}{t^2 - 1}$ .
2. Show that the sum of two continuous functions is continuous.
3. If  $\text{Lt}_{x \rightarrow c} f(x) = 5$  and  $\text{Lt}_{x \rightarrow c} g(x) = -2$ , find  $\text{Lt}_{x \rightarrow c} 2f(x)g(x)$ .
4. Find  $K'(-1)$  if  $k(z) = \frac{1-z}{2z}$ .
5. If  $p = \frac{1}{\sqrt{q+1}}$ , find  $\frac{dp}{dq}$ .
6. What is the derivative product rule?
7. Does differentiability of a function imply continuity? Is the converse true?
8. Find  $\text{Lt}_{x \rightarrow -\infty} \frac{\sqrt{7e}}{x^3}$ .
9. Find  $\text{Lt}_{x \rightarrow +\infty} \frac{3x+7}{x^2-2}$ .
10. Is the x-axis an asymptote of  $f(x) = \frac{1}{x}$ ?

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11. Give an example of a non-integrable function on  $[0, 1]$ .
12. Find the area of the region between  $y = 3x^2$  and the  $x$ -axis on  $[0, b]$ .

(12 × 1 = 12 marks)

## Section B

Answer any nine questions.  
Each question carries 2 marks.

13. Find  $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$ .

14. Prove that  $\lim_{x \rightarrow x_0} x = x_0$ .

15. Prove that the derivative of a constant function is zero.

16. Find the first derivative of  $r = \frac{(\theta - 1)(\theta^2 + \theta + 1)}{\theta^3}$ .

17. Find the first four derivatives of  $y = x^3 - 7x^2 + 8x - 9$ .

18. Find  $\lim_{x \rightarrow \infty} \frac{2x^3 + 7}{x^3 - x^2 + x + 7}$ .

19. Find a linearization of  $f(x) = \sqrt{2+x}$  at  $x = 1$ .

20. Find  $dy$  if  $y = x^3 - 3\sqrt{x}$ .

21. If  $\int_1^2 f(x) dx = -4$ ,  $\int_1^5 f(x) dx = 6$  and  $\int_1^5 g(x) dx = 8$ , find  $\int_2^5 f(x) dx$  and  $\int_1^5 [4f(x) - g(x)] dx$ .

22. Define a continuous function.

23. Find the tangent to  $y^2 = x$  at  $(0, 0)$ .

24. State the first derivative test.

(9 × 2 = 18 marks)

## Section C

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Answer any six questions.  
Each question carries 5 marks.

25. Applying the definition of limit, show that  $\text{Lt}_{x \rightarrow 1} \left( \frac{3x-1}{2} \right)$  is  $\frac{1}{2}$ .
26. Show that the line  $y = mx + b$  is its own tangent at any point  $(x_0, mx_0 + b)$ .
27. Graph the curve  $y = \frac{1}{x}$ . Find its slope at  $x = a \neq 0$ . When is the slope equal to  $-\frac{1}{4}$ ?
28. Find  $\text{Lt}_{x \rightarrow \infty} x \sin \left( \frac{1}{x} \right)$ .
29. Find the total area between the region  $y = -x^2 - 2x$ ,  $-3 \leq x \leq 2$  and the  $x$ -axis.
30. Show that  $y = |x|$  is not differentiable at the origin.
31. Show that if  $n$  is a positive integer,  $\frac{d}{dx} (x^n) = nx^{n-1}$ .
32. State the Mean Value Theorem for definite integrals. Also show that if ' $f$ ' is continuous on  $[a, b]$ ,  $a \neq b$  and if  $\int_a^b f(x) dx = 0$ , then  $f(x) = 0$  at least once in  $[a, b]$ .
33. State and prove the fundamental theorem of calculus for the evaluation of definite integrals using anti-derivatives.

(6 × 5 = 30 marks)

## Section D

Answer any two questions.  
Each question carries 10 marks.

34. (i) Find an equation for the tangent to the curve  $y = x + \frac{2}{x}$  at  $(3, 4)$ .
- (ii) Find the second derivative of  $p = \frac{q^3 + 3}{12q} \left( \frac{q^4 - 1}{q^8} \right)$ .
- (iii) At time ' $t$ ', the position of a body moving along the  $s$ -axis is  $s = t^3 - 6t^2 + 9t$  metres. Find the body's acceleration each time the velocity is 0.

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