

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)*Answer all twelve questions.*

1. At what points are function $f(x) = \frac{1}{(x+2)^2} + 4$ continuous ?
2. Define critical point of a function.
3. Suppose $\lim_{x \rightarrow c} f(x) = 5$ and $\lim_{x \rightarrow c} g(x) = -2$. Find $\lim_{x \rightarrow c} f(x)g(x)$.
4. Find the norm of the partition $[0, 1.2, 1.5, 2.3, 2.6, 3]$.
5. Find absolute minima of $y = x^3$ on $(0, 2]$.
6. Find the interval in which $y = x^3$ is concave up.
7. $\frac{d}{dx} \int_a^x f(t) dt =$ _____.
8. Find dy if $y = x^5 + 37x$.
9. Define average value of a function f on $[a, b]$.
10. Find $\lim_{x \rightarrow \infty} \frac{\pi\sqrt{3}}{x^2}$.
11. Define horizontal asymptote of the graph of a function.
12. Find $\lim_{x \rightarrow 2} \frac{3-x}{3+x}$.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)*Answer any nine questions.*

13. If $2 - x^2 \leq g(x) \leq 2 \cos x$ for all x , find $\lim_{x \rightarrow 0} g(x)$.
14. If $\lim_{x \rightarrow 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \rightarrow 4} f(x)$.
15. Find the derivative of $y = \sqrt{x}$ for $x > 0$. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.
16. Area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4} D^2$. How fast is the area changing with respect to the diameter when the diameter is 10 m?
17. Find absolute extreme values of $g(t) = 8t - t^4$ on $[-2, 1]$.
18. Show that $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$.
19. The radius r of a circle increases from $r_0 = 10$ m to 10.1 m. Estimate the increase in the circle's area A by calculating dA . Compare this with true change ΔA .
20. Find a lower bound for the value of $\int_0^1 \cos x \, dx$ using the inequality $\cos x \geq 1 - x^2/2$.
21. Use Max-Min inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2} \, dx$.
22. Find the area of the region between $y = 4 - x^2$, $0 \leq x \leq 3$ and the x -axis.
23. Find the function with derivative $f'(x) = 2x - 1$ passing through the point $P(0, 0)$.
24. Find $\frac{d}{dx} \int_0^{x^4} \sqrt{u} \, du$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)*Answer any six questions.*

25. Find the slope of the curve $y = 1/x$ at $x = a$. Where does the slope equal $-1/4$? What happens to the tangent to the curve at the point $(a, 1/a)$ as a changes?
26. Show that functions with zero derivatives are constant.
27. Find the asymptotes of the graph of $f(x) = \frac{-8}{x^2 - 4}$.
28. Find $\lim_{x \rightarrow 0} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.
29. Show that functions with the same derivative differ by a constant.
30. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about the x -axis.
31. Express the solution of the initial value problem $\frac{ds}{dt} = f(t)$, $s(t_0) = s_0$ in terms of integral.
32. Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x) dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.
33. Show that if f has a derivative at $x = a$ then f is continuous at a .

(6 × 5 = 30 marks)

Part D (Essay Type)*Answer any two questions.*

34. Find the intervals on which $g(x) = -x^3 + 12x + 5$, $-3 \leq x \leq 3$ is increasing and decreasing. What are the critical points? When does the function assume extreme values and what are these values?
35. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}y^3$, $x = 0$, $y = -1$, $y = 1$ about x -axis.

Turn over

36. Let $f(x) = \begin{cases} 3-x, & x < 2; \\ 2, & x = 2; \\ \frac{x}{2} + 1, & x > 2 \end{cases}$

- (a) Find $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$. and $f(2)$.
- (b) Does $\lim_{x \rightarrow 2} f(x)$ exist? If so, what is it? If not, why not?
- (c) Find $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$.
- (d) Does $\lim_{x \rightarrow 2} f(x)$ exist? If so, what is it? If not, why not?

(2 × 10 = 20 marks)