

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

Complementary Course—Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A*Answer all twelve questions.**Each question carries 1 mark.*

1. Suppose $\lim_{x \rightarrow c} f(x) = 5$ and $\lim_{x \rightarrow c} g(x) = -2$, find $\lim_{x \rightarrow c} 2f(x)g(x)$.
2. Find dy if $y = \cos 3x + x^4$.
3. Write the sum without sigma notation : $\sum_{k=1}^1 0k^2 + 3k$.
4. Find the interval in which the function $y = x^3$ is concave up.
5. Find absolute extrema of $y = x^2$ on $(0, 2)$.
6. Find $\lim_{x \rightarrow -1} \frac{-1}{(1-x)^3}$.
7. Define vertical asymptote.
8. $\frac{d}{dx} \int_{-\pi}^x \cos t dt = \dots\dots$
9. Express the limit $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n \frac{1}{1-c_k} \Delta x_k$ where P is the partition of $[2, 3]$ as an integral.
10. State Mean Value Theorem.

Turn over

11. Find all possible functions with derivative $y' = x^3$.
12. Shortest interval length of a partition is called _____.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** questions.
Each question carries 2 marks.

13. Show that if $\lim_{x \rightarrow c} |f(x)| = 0$, then $\lim_{x \rightarrow c} f(x) = 0$.
14. If $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \rightarrow 0} f(x)$.
15. If $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \rightarrow -2} \frac{f(x)}{x}$.
16. Find the slope of the curve $y = 1/x$ at $x = a$.
17. Differentiate $f(x) = \frac{x}{x-1}$. Where does the curve $y = f(x)$ have slope -1 ?
18. Show that $y = -x$ is not differentiable at $x = 0$.
19. Find the equation for the tangent to the curve $y = x + \frac{2}{x}$ at $(1, 3)$.
20. Find absolute extrema of $y = x^{2/3}$ on $[-2, 3]$.
21. Find the function whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.
22. Show that $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$.
23. Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x) dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.
24. Express the solution of the initial value problem $\frac{dy}{dx} = \tan x$, $y(1) = 5$ as an integral.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

*Answer any six questions.
Each question carries 5 marks.*

25. Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.
26. When does a function not have a derivative at a point? Explain.
27. Show that if f has a derivative at $x = c$, then f is continuous at $x = c$.
28. Show that functions with same derivatives differ by a constant.
29. Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.
30. Find $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 7}$.
31. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \leq x \leq 2$ about the x -axis.
32. Find $\lim_{h \rightarrow 0^+} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.
33. Define $f(3)$ in a way that extends $f(x) = \frac{x^2 - 9}{x - 3}$ to be continuous at $x = 3$.

(6 × 5 = 30 marks)

Part D (Essay Type)

*Answer any two questions.
Each question carries 10 marks.*

34. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.
35. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}y^2, x = 0, y = -1, y = 1$ about x -axis.

Turn over

36. Let $f(x) = \begin{cases} \sqrt{1-x^2}, & 0 \leq x < 1; \\ 1, & 1 \leq x < 2; \\ 2, & x = 2. \end{cases}$

- (a) What are the domain and range of f ?
- (b) At what points c , if any, does $\lim_{x \rightarrow c} f(x)$ exist?
- (c) At what points does only the left-hand limit exist?
- (d) At what points does only the right-hand limit exist?

(2 × 10 = 20 marks)