

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all twelve questions.

1. Find $\lim_{x \rightarrow 1} \frac{-1}{(3x-1)^2}$.
2. If f is the identity function, then find $\lim_{x \rightarrow x_0} f(x)$ where x_0 is a point in the domain of $f(x)$.
3. Find $\lim_{x \rightarrow \infty} \left(5 + \frac{1}{x}\right)$.
4. Differentiate $\sin(2x+3)$ with respect to x .
5. Find the second derivative of $y = -x^2 + 3$.
6. Define absolute maximum of a function.
7. Define an increasing function.
8. State the Mean Value theorem.
9. Find the critical points of $y = x^{2/3}(x+2)$.
10. Evaluate $\sum_{k=1}^7 -2k$.
11. Give an example of a function which is not Riemann integrable.
12. The volume of a solid of cross-section area $A(x)$ from $x = a$ to $x = b$ is

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)Answer any **nine** questions.

13. Show that if $\lim_{x \rightarrow c} |f(x)| = 0$, then $\lim_{x \rightarrow c} f(x) = 0$.
14. Evaluate $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$.
15. For the function $f(x) = \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ find $\lim_{x \rightarrow 0} f(x)$ or explain why they do not exist.
16. Show that the derivative of a constant function is zero.
17. Find the derivative of $y = \frac{x^2 - 1}{x^2 + 1}$.
18. Find the linearization of $f(x) = \sqrt{1+x}$ at $x=0$.
19. Find the absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.
20. Evaluate $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$.
21. Express the limit $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n (3c_k^2 - 2c_k + 5) \Delta x_k$ as definite integrals, where P is a partition of $[-3, 5]$.
22. Find $\frac{dy}{dx}$ if $y = \int_0^{x^2} \cos t \, dt$.
23. Using the inequality $\cos x > \left(1 - \frac{x^2}{2}\right)$, find a lower bound for the value of $\int_0^1 \cos x \, dx$.
24. State and prove the Mean Value theorem for definite integrals.

(9 × 2 = 18 marks)

Part C (Short Essay Type)*Answer any six questions.*

25. Find the continuous extension to $x = 2$ of the function $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$.
26. Let $f(x) = \begin{cases} 0 & x \leq 0 \\ \sin \frac{1}{x} & x > 0 \end{cases}$
- (a) Does $\lim_{x \rightarrow 0^+} f(x)$ exist? If so what is it? If not, why not?
- (b) Does $\lim_{x \rightarrow 0^-} f(x)$ exist? If so what is it? If not, why not?
- (c) Does $\lim_{x \rightarrow 0} f(x)$ exist? If so what is it? If not, why not?
27. Find the equation of the tangent to the curve $f(x) = (x + 1)^2$ at the point (1, 4).
28. If $s = t^2 - 3t + 2$, $0 \leq t \leq 2$ gives the positions of a body moving on a coordinate line, with s in metres and t in seconds.
- (a) Find the body's displacement and average velocity for the given time interval.
- (b) Find the body's speed and acceleration at the endpoints of the interval.
29. Show that $\lim_{x \rightarrow 0^+} (1 + x)^{1/x} = e$.
30. Find the asymptotes of the curve $y = \frac{x + 3}{x + 2}$.
31. Find the intervals on which $g(x) = -x^3 + 12x + 5$, $-3 \leq x \leq 3$ is increasing and decreasing. Where does the function assume extreme values?
32. A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.
33. Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$. At what point in the interval does this function assume its average value?

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any two questions.

34. If $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$, prove that $\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$.
35. The first derivative of the continuous function $y = f(x)$ is $y' = 2 + x - x^2$. Find y'' and sketch the general shape of the graph of f .
- What are the critical points of f ?
 - On what intervals is f increasing or decreasing?
 - At what point if any, does f assume local maximum and minimum values?
36. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$.

(2 × 10 = 20 marks)