51510 (Pages : 3) Name..................................

Reg. No..................................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(U.G.—CCSS)

(Complementary Course)

MM 3C 03—MATHEMATICS

ne : Three Hours Maximum : 30 Weightage

I. Objective Type Questions : Answer *all* questions :

Each question of weightage ¼.

1 Check for exactness :

$\left(x^3 + 3xy^2\right) dx + \left(3x^2y + y^3\right) dy = 0.$

2 Curves that intersect a given curve at right angels are called ——————.

3 Solve $y' = \log x$.

4 When is a square matrix A said to be non-singular ?

5 Find the characteristic roots of $\begin{bmatrix} 1 & 3 \\ 10 & 2 \end{bmatrix}$.

6 If '2' is an eigenvalue of a square matrix A, give one root of A^T.

7 Define a solenoidal vector.

8 When are two vectors $\vec{a}$ and $\vec{b}$ said to be orthogonal ?

9 What is the divergence of $\vec{a} = \left[x^2, y^2, z^2\right]$?

10 Find a unit normal vector to the surface $S : x^2 + y^2 + z^2 = a^2$.

11 Give the parametric representation of the plane $3x + 2y + z = 6$.

12 If $\vec{F} = \text{grad } f,\ f = x^2 + y^2 + 2z^2$, find $\int_C \vec{F} \text{ od } \vec{r}$ where C has initial point $A(0,0,0)$ and terminal point $B : (2, 2, 2)$.

(12 × ¼ = 3 weightage)

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II. Short Answer Type Questions : Answer *all* questions.

Each question of weightage 1.

13 Find the rank of the following matrix

$$A = \begin{bmatrix} 4 & 7 & 6 \\ 8 & 14 & 12 \\ 1 & 1 & 1 \end{bmatrix}.$$

14 State Cayley Hamilton Theorem.

15 Solve $\left(1+x^2\right)\dfrac{dy}{dx} = 1 + y^2,\ y(0) = 1.$

16 Find an integrating factor for $2x \tan y\, dx + \sec^2 y\, dy = 0$.

17 Find the work done by $\vec{p} = [2,\ 6,\ 6]$ if it displaces a body from $A:(3,\ 4,\ 0)$ to $B:(5,\ 8,\ 0)$.

18 Prove that $\operatorname{div}[\operatorname{grad} f] = \nabla^2 f$.

19 Find the tangential and normal accelerations of $\vec{r}(t) = [b\cos t, b\sin t, c]$.

20 Check for path independence :

$\sinh xz\,(z dx - xz dz)$.

21 Use Green's Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \operatorname{grad}(\sin x \cos y)$ and C is the ellipse $25x^2 + 9y^2 = 225$.

(9 × 1 = 9 weightage)

III. Short Essay or Paragraph Questions : Answer any *five* questions :

Each question of weightage 2.

22 Solve $2xyy' = y^2 - x^2$.

23 Find the rank by reducing to normal form :

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}.$$

24 Find the eigenvalues of $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

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25 Find the directional derivative of $f = \left(x^2 + y^2 + z^2\right)^{-1/2}$ at $(3, 0, 4)$ along $\vec{a} = [1, 1, 1]$.

26 Find the length of the hypocycloid $\bar{r}(t) = a\cos^3 t\,\hat{i} + a\sin^3 t\,\hat{j}$.

27 Test for exactness and hence evaluate :

$$\int_{(3,\,3/2)}^{(4,\,1/2)} 2x\sin\pi y\,dx + \pi x^2\cos\pi y\,dy.$$

28 Evaluate using the Gauss Divergence Theorem : $\iint_S \vec{F} \cdot \vec{n}\,dA$, $\vec{F} = \left[x^3, y^3, z^3\right]$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 9$.

(5 × 2 = 10 weightage)

IV. Essay Questions : Answer any *two* questions.
Each question of weightage 4.

29 Find the Orthogonal Trajectories of $y = c\sqrt{x}$.

30 Use Cayley Hamilton Theorem to find A^3 and A^4 if $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

31 Verify Stokes' Theorem for $\vec{F} = [y, z, x]$ and S is the portion of the paraboloid $z = 1 - \left(x^2 + y^2\right)$, $z \geq 0$.

(2 × 4 = 8 weightage)

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