

**FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018**

(CUCBCSS—UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 80 M

**Part A (Objective Type)**

*Answer all questions.*

*Each question carries 1 mark.*

1. What do you mean by fundamental period ?
2. Write the characteristic equation of the differential equation  $2y'' + 10y' + 25y = 0$ .
3. Write the 2 dimensional wave equation.
4. Give an example of a function which is neither odd nor even.
5. State the existence and uniqueness theorem for initial value problem.
6. Find the Laplace transform of  $f'(t)$ .
7. Find the Wronskian of  $e^{\lambda x}$  and  $xe^{\lambda x}$ .
8. Find  $L^{-1}\left(\frac{1}{s/a - 1}\right)$ .
9. Find  $L(t + e^t)$ .
10. State the second shifting theorem of Laplace transform.
11. Write the error estimate for Simpson's rule.
12. Write the formula for Runge-Kutta method.

(12 × 1 = 12 m)

**Part B (Short Answer Type)**

*Answer any nine questions.*

*Each question carries 2 marks.*

13. Find the Fourier cosine series of  $f(x) = \pi - x$ ,  $0 < x < \pi$ .
14. Show that  $u = e^{-w^2 c^2 t} \sin wx$  is a solution of heat equation.
15. Apply Picard's iteration to solve  $y' = y - x^2$ ,  $y(0) = 1$ . Also find  $y(0, 1)$  and  $y(0, 2)$ .
16. By reducing to first order solve  $y'' + (y')^2 = 0$ .

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17. Find fundamental set of solutions of  $2t^2y'' + 3ty' - y = 0$   $t > 0$ . Given that  $y_1(t) = t^{-1}$  is a solution.
18. Solve  $y'' + 8y' + 16y = 0$ .
19. Find the inverse transform of  $\frac{4}{(s+1)(s+2)}$ .
20. Solve the initial value problem  $y'' + y' - 6y = 1$   $y(0) = 0$ ,  $y'(0) = 1$ .
21. Find  $L(te^{-t} \sin t)$ .
22. Using convolution property find  $L^{-1}\left[\frac{1}{s(s^2 + 4^2)}\right]$ .
23. Use Trapezoidal rule with  $n = 4$  to estimate  $\int_1^2 x^2 dx$ .
24. Solve the partial differential equation  $u_{yy} + 4u = 0$ .

(9 × 2 = 18 marks)

**Part C (Short Essays)**

Answer any six questions.  
Each question carries 5 marks.

25. Solve  $(x^2D^2 + 3xD + 1)y = 0$ .
26. Find the general solution of the differential equation  $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$ .
27. Find the Laplace transform of  $e^{-2t}u(t-3)$ .
28. Find  $L^{-1}\left(\frac{se^{-2s}}{s^2 + \pi^2}\right)$ .
29. Find the Fourier series expansion of:

$$f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \pi < x < 2\pi \end{cases}$$

30. Solve  $xy'' - y' = (3+x)x^2e^x$ .
31. Find the solution of  $u_x + u_y = 0$  by separating variables.
32. Find approximate solution of  $y' + y = e^x$   $y(0) = 0$ .
33. Obtain the half-range cosine series of  $f(x) = x$  when  $0 < x < 2$ .

(6 × 5 = 30 marks)

## Part D

Answer any two questions.  
Each question carries 10 marks.

34. Solve the non-homogeneous equation :  $y'' - y' - 2y = 10 \cos x$ .
35. Solve the initial value problem  $y'' + 2y' + 2y = r(t)$

$$\text{where } r(t) = \begin{cases} 10 \sin 2t & \text{if } 0 < t < \pi \\ 0 & \text{if } t > \pi \end{cases}$$

$$y(0) = 1 \quad y'(0) = -5.$$

Using Laplace transforms.

36. Find the Fourier series of the functions

$$f(x) = \begin{cases} x + x^2 & -\lambda < x < \pi \\ \pi^2 & x = \pm \pi \end{cases}$$

Deduce that  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$ .

(2 × 10 = 20 marks)