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Name.............................................

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# FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

## (CUCBCSS–UG)

### Mathematics

### MAT 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

## Part A (Objective Type)

*Answer all the **twelve** questions.*
*Each question carries 1 mark.*

1. State the Existence and Uniqueness theorem for initial value problem.

2. Define and give an example of an even function.

3. What do you mean by a non-linear differential equation ?

4. Solve $y'' - y' - 2y = 0$.

5. Define a unit step function.

6. State the existence theorem for Laplace transforms.

7. Find $L^{-1}\left(\dfrac{a}{s^2 - a^2}\right)$.

8. Find $L\left(t^{-\frac{1}{2}}\right)$.

9. Define a rectangular wave.

10. Write the 2-dimensional Poisson equation.

11. Give a formula for an error for Simpson's rule.

12. Write the formula for Runge Kutta method.

$(12 \times 1 = 12 \text{ marks})$

## Part B (Short Answer Type)

*Answer any **nine** questions.*
*Each question carries 2 marks.*

13. Find the particular integral for $y' + 4y = 8x^2$.

14. Find a basis for the solution of the differential equation $y'' + y = 0$.

**Turn over**

15. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at}F(t)$.

16. Solve $3y'' - 8y' - 3y = 0, y(-3) = 1, y(3) = \left(\dfrac{1}{e^2}\right)$.

17. Find $L\left(e^{-\alpha t}\cos\beta t\right)$.

18. If $f(x)$ is a periodic function of $x$ of period $p$, show that $f(ax), a \neq 0$, is a periodic function of $x$ of period $\dfrac{p}{a}$.

19. Find the Fourier series of $f(x) = x + |x|, -\pi < x < \pi$.

20. Show that $u = e^{-t}\sin x$ is a solution of heat equation.

21. Apply Picards iteration to solve $y' = y - x^2, y(0) = 1$ also find $y(0.1)$ and $y(0.2)$.

22. Evaluate $\displaystyle\int_{-3}^{3} x^4\, dx$ using Simpson's rule.

23. What do you mean by convolution ?

24. Evaluate $\displaystyle\int_{0}^{6} \dfrac{1}{1+x}\, dx$ by Trapezoidal rule.

(9 × 2 = 18 marks)

## Part C (Short Essays)

*Answer any six questions.*
*Each question carries 5 marks.*

25. Solve $\left(4x^2 D^2 + 12xD + 3\right)y = 0$.

26. Find a general solution of the differential equation $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$.

27. Find the Laplace transform of $(t-1)^2 u(t-1)$.

28. Find $L^{-1}\left(\dfrac{4\left(e^{-2s}-2e^{-5s}\right)}{s}\right)$.

29. Solve $u_{xy}=u_x$.

30. Find the Fourier series of $f(x)=\begin{cases}-k, \text{ if } -\pi<x<0; \\ k, \text{ if } 0<x<\pi,\end{cases}$ and $f(x+2\pi)=f(x)$.

31. Given $y'=-y$, $y(0)=1$. Find the value of $y$ at $x=(0.01)(0.01)(0.04)$ by improved Euler method.

32. Find approximate solution to $y'+y=e^x$, $y(0)=0$.

33. Evaluate $\displaystyle\int_{4}^{5.2}\log_e x\,dx$ using Simpson's rule.

$(6 \times 5 = 30 \text{ marks})$

## Part D

*Answer any **two** questions.*
*Each question carries 10 marks.*

34. Solve $x^2y''-2xy'+2y=x^3\sin x$.

35. Solve the integral equation $y(t)=t+\displaystyle\int_{0}^{t}y(\tau)\sin(t-\tau)\,d\tau$.

36. Find the Fourier series $f(x)=\begin{cases}x+x^2, \text{ if } -\pi<x<\pi; \\ \pi^2, \text{ if } x=\pm\pi.\end{cases}$

$(2 \times 10 = 20 \text{ marks})$