C 31164 (Pages : 4) Name..............................

Reg. No..............................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **ten** *questions.*
Each question carries 1 *mark.*

1. Verify that $y = c \sec x$ is a solution of $y' = y \tan x$.

2. Solve $y' = -ky^2$.

3. Test for exactness : $\sinh x \cos y \, dx - \cosh x \sin y \, dy = 0$.

4. Define rank of a matrix.

5. Find the characteristic roots of $A = \begin{bmatrix} -11 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$.

6. State Cayley-Hamilton Theorem.

7. If $\vec{p}, [1, \ -1, \ 3]$ and $[-3, \ 2 \ \ 4]$ are in equilibrium, find $\vec{p}$.

8. Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$.

9. Illustrate commutativity of the vector dot product with an example.

10. Define a simply connected domain.

11. State Green's Theorem in the plane.

12. Find the unit normal vector to the sphere $x^2 + y^2 + z^2 = a^2$.

($10 \times 1 = 10$ marks)

Section B

Answer any **ten** *questions.*
Each question carries 2 *marks.*

13. Solve $e^y\, dx + (xe^y + 2y)\, dy = 0$.

14. Solve $y' - y = e^{2x}$.

15. Find the characteristic equation of $A = \begin{bmatrix} 8 & -1 \\ 2 & 8 \end{bmatrix}$.

16. Solve $\frac{dy}{dx} + \frac{y}{x} = x$.

17. Using Cayley-Hamilton Theorem, find A^2 if $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

18. Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \end{bmatrix}$.

19. Find a parametric representation of the straight line through $(2, 3, 0)$ and $(5, -1, 0)$.

20. Find a tangent vector and unit tangent vector for $\bar{r}(t) = [2\cos t, 2\sin t, 0]$.

21. Show that the form under the integral sign is exact $\int 2xy^2 dx + 2x^2 y dy + dz$.

22. Give the standard form of the Bernoulli Equation with an example.

23. Use Green 's Theorem to find the area enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

24. Find the curl of $[2x,\ 4y,\ 8z]$.

(10 × 2 = 20 marks)

Section C

Answer any **six** *questions.*

Each question carries 5 *marks.*

25. Find an integrating factor and solve : $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$.

26. Find the Orthogonal Trajectories of $y = cx^{3/2}$.

27. Reduce to normal form and find the rank of $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 4 & -6 & -10 \\ 5 & 8 & -12 & -19 \end{bmatrix}$.

28. Solve the system of equations
$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0.$$

29. Verify Cayley-Hamilton Theorem for $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

30. Find the length of $\bar{r}(t) = \left[a\cos^3 t, a\sin^3 t, 0\right]$ from $t = 0$ *to* $t = \frac{\pi}{2}$.

31. (i) Prove that for any twice differentiable scalar function 'f', curl (grad f) $= \vec{0}$.

 (ii) Prove that for any vector $\vec{v}$, div (curl $\vec{v}$) $= 0$.

32. State Gauss Divergence Theorem and use it to evaluate $\iint_S \overline{F} \circ \bar{n}\; dA$, $\overline{F} = \left[x^3, \; y^3, \; z^3\right]$, S is the surface $x^2 + y^2 + z^2 = 16$.

33. Find the work done by $\overline{F} = \left[e^x, \; e^{-y}, \; e^z\right]$, c is the curve $[t, t^2, t]$ from $(0, 0, 0)$ to $(1, 1, 1)$.

(6 × 5 = 30 marks)

Turn over

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the characteristic roots and characteristic vectors of $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$.

35. (i) Find the tangential and normal acceleration of $\bar{r}(t) = [2\cos t, 2\sin t, 3]$.

(ii) Find the directional derivative of $f(x, y, z) = xyz$ at $(-1, 1, 3)$ in the direction of $[1, -2, 2]$.

(iii) If $\vec{v} = \text{grad } f$ find 'f' for $\vec{v} = [yz, zx, xy]$.

36. State Stoke's Theorem and verify it for $\bar{F} = [x^2 - y^2, 2xy, 0]$, S is the surface of the rectangle $x = 0, y = 0, x = a, y = b$.

(2 × 10 = 20 marks)