D 51292 (Pages : 3) Name.....................................

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# THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Part A (Objective Type)

*Answer* **all the twelve** *questions.*
*Each question carries* 1 *mark.*

1. Write the general form of Bernoulli's differential equation.
2. Solve $y' = \frac{xy}{2}$.
3. What is the degree of the differential equation $x^3 y''' y' + 2e^x y'' = 0$.
4. State Cayley-Hamilton theorem.
5. What is the determinant of a 2 × 2 matrix whose rank is 1 ?
6. What is the normal form of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{pmatrix}$ ?
7. Find the resultant of the vectors $p = [4, -2, -3]$, $q = [8, 8, 0]$.
8. Write the parametric representation of a straight line through the point (4, 2, 0) and in the direction of the vector $i + j$.
9. Define gradient of a function.
10. Find the directional derivative of $f = x - y$ at (4, 5) in the direction of $2i + j$.
11. Define a smooth curve.
12. State Green's theorem in plane.

(12 × 1 = 12 marks)

## Part B (Short Answer Type)

*Answer any* **nine** *questions.*
*Each question carries* 2 *marks.*

13. Write the condition for the differential equation $Mdx + Ndy = 0$ become exact. What is the form of its solution ?
14. Find an integrating factor for $2xydx + 3x^2dy = 0$ and solve it.

**Turn over**

15. Find the characteristic roots of the matrix $\begin{pmatrix} 4 & 3 & 1 \\ -4 & -3 & -1 \\ 1 & 2 & 5 \end{pmatrix}$.

16. Write the elementary transformations in a matrix.

17. Find the component of $a = [4, 0, -3]$ in the direction of $b = [1, 1, 1]$.

18. Find the arc length parameter for the helix $r(t) = [a\cos t, a\sin t, ct]$.

19. Find div $v$ where $v = x^2 i + y^2 j + z^2 k$.

20. Define Jacobian.

21. Show that $\operatorname{curl}(u+v) = \operatorname{curl} u + \operatorname{curl} v$.

22. Find $\nabla^2 f$ where $f = e^{2x} \sin 2y$.

23. Show that $\int_{(-1,5)}^{(4,3)} 3z^2 dx + 6xzdz$ is path independent.

24. Write the formula for finding the area of a plane region as a line integral over the boundary.

(9 × 2 = 18 marks)

## Part C (Short Essay)

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Solve $(2x - 4y + 5)y' + (x - 2y + 3) = 0$.

26. Solve $2x \tan y dx + \sec^2 y dy = 0$.

27. Find the eigen values and eigen vectors corresponding to any one eigen value of the matrix :

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

28. Use Cayley-Hamilton theorem to find $A^{-1}$ and $A^3$ where A is $\begin{pmatrix} 1 & 5 \\ 3 & 8 \end{pmatrix}$.

29. Find the speed and tangential acceleration of an object moving along the curve :

$r(t) = \cos t i + \sin 2t j + \cos 2t k$.

30. Find unit normal vectors for the surface $z = \sqrt{(x^2 + y^2)}$ at (6, 8, 10).

31. Find the volume of the region in space bounded by the co-ordinate planes and the surfaces $y = 1 - x^2, z = 1 - x^2$.

32. Find the area of the region in the first quadrant bounded by the cardioid $r = a(1 + \cos\theta)$.

33. Verify Greens theorem in the plane for $F = [3y^2, x - y^4]$ and the region is the rectangle with vertices (1, 1), (–1, 1), (–1, –1), (1, –1).

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Test for consistency and solve the following system :

(a) $2x + y + z = 5$
$x - y = 0$
$2x + y - z = 1$

(b) $x + 2y + 3z = 14$
$2x - y + 5z = 15$
$-3x + 2y + 4z = 13$.

35. Solve (a) $2\sin(y^2)\,dx + xy\cos(y^2)\,dy = 0, y(2) = \sqrt{\frac{\pi}{2}}$; (b) Find the angle between $3x + 5y = 0$ and $4x - 2y = 1$.

36. Verify Stoke's theorem for $F = [y^2, -x^2, 0]$ over the circular semi-disk $x^2 + y^2 \le 4, y \ge 0, z = 0$.

(2 × 10 = 20 marks)