D.9515

Name: .....

Reg.No. .....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2010

MATHEMATICS (COMPLEMENTARY COURSE)
MM.IC.01 – Mathematics

Time: Three hours

30 Weightage

Part A : Objective type questions

(Answer all 12 questions. Each bunch of four questions carries 1 Weight)

1. Evaluate $\lim_{y\to -5} \frac{y^2}{5-y}$.
2. Find $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x}$.
3. Determine $\lim_{x\to 1+} \sqrt{\frac{x-1}{x+2}}$.
4. Find the point of discontinuity of the function $y = \frac{x+2}{cosx}$.

$(1 \times 1/4 = 1 \text{ Weight})$

5. Find the slope of the curve $f(x) = \frac{x}{x-2}$ at (3,3).
6. Determine the point(s) at which the curve $y = x^3 + \frac{4}{3}x^2 - 5x + 1$ has horizontal tangents
7. Find the derivative of $y = (x^2 + 1)(x^3 + 2)$.
8. State the quotient rule of differentiation.

$(1 \times 1/4 = 1 \text{ Weight})$

9. Find the absolute maximum of $y = x^2$ in [0,2].
10. Where does the function $y = secx$ have vertical asymptotes?
11. Evaluate $\sum_{k=1}^{1} \frac{k-1}{k}$.
12. Find the average value of $f(x) = 4 - x^2$ on [0, 3].

$(1 \times 1/4 = 1 \text{ Weight})$

Short answer type questions

(Answer all 9 questions. Each question carries one Weight)

13. Find the average rate of change of the function $f(x) = x^3 + 1$ in the interval [2,3].
14. If $1 - \frac{x^2}{4} \le u(x) \le 1 + \frac{x^2}{2}$ for all $x \neq 0$, find $\lim_{x\to 0} u(x)$.
15. Show that $\lim_{x\to a} k = k$.
16. Check the continuity of the function $f(x) = |x|$ in R.

17. Find the second order derivative of $y = \frac{(x-1)(x^2-2x)}{x^4}$.

18. Check whether the function $f(x) = \sqrt{x(x-1)}$ satisfy the hypotheses of Mean Value Theorem in [0,1].

19. Find the critical points of $f(x) = x^{1/3}(x-4)$.

20. Determine the interval on which the function $f(x) = -x^3 + 12x + 5$, $x\varepsilon[-3,3]$ is increasing.

21. Find the area of the region between the curve $y = 3x^2$ and the x-axis on the interval [0, b].

(9 x 1 = 9 Weights)

Short essay questions

(Answer any 5 questions. Each question carries two Weights)

22. At t seconds after lift-off, the height of a rocket is $3t^2$ ft. How fast is the rocket climbing after 10 sec?

23. Can $\lim_{x\to c} {f(x)}/{g(x)}$ exist even if $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$? Give reasons for your answer.

24. Explain why the function $f(x) = sin\left(\frac{1}{x}\right)$ has no continuous extension to x = 0.

25. Find $\frac{dy}{dx}$, if $y = 2x^3$ using the definition of derivatives.

26. The curves $y = x^2 + ax + b$ and $y = cx - x^2$ have a common tangent line at the point (1, 0). Find a, b, and c.

27. Suppose the derivative of the function $y = f(x)$ is $y' = (x-1)^2(x-2)(x-4)$. At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection?

28. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines y = 1, x = 4 about the line y = 1.

(5 x 2 = 10 Weights)

[Turn Over]

Essay questions

(Answer any 2 questions. Each question carries 4 Weights)

29. Graph the function

$$f(x) = \begin{cases} \sqrt{1-x^2} & if\ 0 \le x \le 1 \\ 1 & if\ 1 \le x \le 2 \\ 2 & if\ x = 2 \end{cases}$$

a) What are the domain and range of f?

b) At what points c, if any, does $\lim_{x \to c} f(c)$ exist?

e) At what points does only the left-hand limit exist?

d) At what points does only the right-hand limit exist?

e) At what points does the function is continuous?

30. The first derivative of a continuous function $y = f(x)$ is $y' = (x^2 - 2x)(x-5)^2$. Find y" and sketch the general shape of the graph of f.

a) What are the critical points of f?

b) On what intervals is f increasing or decreasing?

c) At what points, if any, does f assume local maximum and minimum values?

d) What are the points of inflexion of f?

31. Find:

a) The length of the curve y = (x /2)2/3 from x = 0 to x = 2.

b) The area of the surface generated by revolving the curve $y = x^3, 0 \le x \le 1/2$, about the x-axis.

c) The volume of the solid generated by revolving the region bounded by the curves $x = \sqrt{y}, x = -y, y = 2$.

(2 x4 = 8 Weights)