D 92865 **(Pages : 4)** **Name..**

Reg. No.......................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. Find $\lim\limits_{x \to 1} \dfrac{-1}{(3x-1)^2}$.

2. If f is the identity function , then find $\lim\limits_{x \to x_0} f(x)$ where x_0 is a point in the domain of $f(x)$.

3. Find $\lim\limits_{x \to \infty} \left(5 + \dfrac{1}{x}\right)$.

4. Differentiate $\sin(2x+3)$ with respect to x.

5. Find the second derivative of $y = -x^2 + 3$.

6. Define absolute maximum of a function.

7. Define an increasing function.

8. State the Mean Value theorem.

9. Find the critical points of $y = x^{2/3}(x+2)$.

10. Evaluate $\sum\limits_{k=1}^{7} -2k$.

11. Give an example of a function which is not Riemann integrable.

12. The volume of a solid of cross-section area A (x) from $x = a$ to $x = b$ is ......

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Show that if $\lim_{x \to c} |f(x)| = 0$, then $\lim_{x \to c} f(x) = 0$.

14. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

15. For the function $f(x) = \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ find $\lim_{x \to 0} f(x)$ or explain why they do not exist.

16. Show that the derivative of a constant function is zero.

17. Find the derivative of $y = \frac{x^2 - 1}{x^2 + 1}$.

18. Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$.

19. Find the absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.

20. Evaluate $\lim_{x \to 0} \frac{3x - \sin x}{x}$.

21. Express the limit $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(3c_k^2 - 2c_k + 5\right) \Delta x_k$ as definite integrals, where P is a partition of $[-3, 5]$.

22. Find $\frac{dy}{dx}$ if $y = \int_0^{x^2} \cos t \, dt$.

23. Using the inequality $\cos x > \left(1 - x^2/2\right)$, find a lower bound for the value of $\int_0^1 \cos x \, dx$.

24. State and prove the Mean Value theorem for definite integrals.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Find the continuous extension to $x = 2$ of the function $f(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$.

26. Let $f(x) = \begin{cases} 0 & x \le 0 \\ \sin \frac{1}{x} & x > 0 \end{cases}$.

 (a) Does $\lim_{x \to 0^+} f(x)$ exist ? If so what is it ? If not, why not ?

 (b) Does $\lim_{x \to 0^-} f(x)$ exist ? If so what is it ? If not, why not ?

 (c) Does $\lim_{x \to 0} f(x)$ exist ? If so what is it ? If not, why not ?

27. Find the equation of the tangent to the curve $f(x) = (x+1)^2$ at the point (1, 4).

28. If $s = t^2 - 3t + 2, 0 \le t \le 2$ gives the positions of a body moving on a coordinate line, with s in metres and t in seconds.

 (a) Find the body's displacement and average velocity for the given time interval.

 (b) Find the body's speed and acceleration at the endpoints of the interval.

29. Show that $\lim_{x \to 0+} (1 + x)^{1/x} = e$.

30. Find the asymptotes of the curve $y = \dfrac{x+3}{x+2}$.

31. Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \le x \le 3$ is increasing and decreasing. Where does the function assume extreme values ?

32. A pyramid 3 *m* high has a square base that is 3 *m* on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x *m* on a side. Find the volume of the pyramid.

33. Find the average value of $f(x) = 4 - x^2$ on [0, 3]. At what point in the interval does this function assume its average value ?

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*

34. If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, prove that $\lim_{x \to c} (f(x) + g(x)) = L + M$.

35. The first derivative of the continuous function $y = f(x)$ is $y' = 2 + x - x^2$. Find y'' and sketch the general shape of the graph of *f*.

    (a) What are the critical points of *f* ?

    (b) On what intervals is *f* increasing or decreasing ?

    (c) At what point if any, does *f* assumes local maximum and minimum values ?

36. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

(2 × 10 = 20 marks)