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Name.............................

Reg. No........................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Section A

Answer all questions.
Each question carries 1 mark.

1. Find $\underset{t \to 1}{\text{Lt}} \dfrac{t^2 + t - 2}{t^2 - 1}$.

2. Show that the sum of two continuous functions is continuous.

3. If $\underset{x \to c}{\text{Lt}}\, f(x) = 5$ and $\underset{x \to c}{\text{Lt}}\, g(x) = -2$, find $\underset{x \to c}{\text{Lt}}\, 2f(x)\, g(x)$.

4. Find $K'(-1)$ if $k(z) = \dfrac{1-z}{2z}$.

5. If $p = \dfrac{1}{\sqrt{q+1}}$, find $\dfrac{dp}{dq}$.

6. What is the derivative product rule ?

7. Does differentiability of a function imply continuity ? Is the converse true ?

8. Find $\underset{x \to -\infty}{\text{Lt}} \dfrac{\sqrt{7e}}{x^3}$.

9. Find $\underset{x \to +\infty}{\text{Lt}} \dfrac{3x + 7}{x^2 - 2}$.

10. Is the x-axis an aymptote of $f(x) = \dfrac{1}{x}$?

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11. Give an example of a non-integrable function on $[0, 1]$.

12. Find the area of the region between $y = 3x^2$ and the x-axis on $[0, b]$.

Section B

*Answer any **nine** questions.*
Each question carries 2 marks.

13. Find $\underset{x \to 9}{Lt} = \dfrac{\sqrt{x} - 3}{x - 9}$.

14. Prove that $\underset{x \to x_0}{Lt}\ x = x_0$.

15. Prove that the derivative of a constant function is zero.

16. Find the first derivative of $r = \dfrac{(\theta - 1)\left(\theta^2 + \theta + 1\right)}{\theta^3}$.

17. Find the first four derivatives of $y = x^3 - 7x^2 + 8x - 9$.

18. Find $\underset{x \to \infty}{Lt}\ \dfrac{2x^3 + 7}{x^3 - x^2 + x + 7}$.

19. Find a linearization of $f(x) = \sqrt{2 + x}$ at $x = 1$.

20. Find dy if $y = x^3 - 3\sqrt{x}$.

21. If $\displaystyle\int_{1}^{2} f(x)\,dx = -4, \int_{1}^{5} f(x)\,dx = 6$ and $\displaystyle\int_{1}^{5} g(x)\,dx = 8$, find $\displaystyle\int_{2}^{5} f(x)\,dx$ and $\displaystyle\int_{1}^{5} [4f(x) - g(x)]\,dx$

22. Define a continuous function.

23. Find the tangent to $y^2 = x$ at $(0, 0)$.

24. State the first derivative test.

Answer any six questions.
Each question carries 5 marks.

25. Applying the definition of limit, show that $\underset{x \to 1}{Lt} \left(\dfrac{3x-1}{2} \right)$ is $\dfrac{1}{2}$.

26. Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

27. Graph the curve $y = \dfrac{1}{x}$. Find its slope at $x = a \neq 0$. When is the slope equal to $-\dfrac{1}{4}$?

28. Find $\underset{x \to \infty}{Lt} \; x \sin \left(\dfrac{1}{x} \right)$.

29. Find the total area between the region $y = -x^2 - 2x, \; -3 \leq x \leq 2$ and the x-axis.

30. Show that $y = |x|$ is not differentiable at the origin.

31. Show that if n is a positive integer, $\dfrac{d}{dx} \left(x^n \right) = nx^{n-1}$.

32. State the Mean Value Theorem for definite integrals. Also show that if f' is continuous on $[a, b]$, $a \neq b$ and if $\displaystyle\int_a^b f(x) \, dx = 0$, then $f(x) = 0$ atleast once in $[a, b]$.

33. State and prove the fundamental theorem of calculus for the evaluation of definite integrals using anti-derivatives.

$(6 \times 5 = 30 \text{ marks})$

Section D

Answer any two questions.
Each question carries 10 marks.

34. (i) Find an equation for the tangent to the curve $y = x + \dfrac{2}{x}$ at $(3, 4)$.

(ii) Find the second derivative of $p = \dfrac{q^3 + 3}{12q} \left(\dfrac{q^4 - 1}{q^3} \right)$.

(iii) At time 't', the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$ metres. Find the body's acceleration each time the velocity is O.