33326 (Pages : 4) Name..............................

Reg. No..............................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

ime : Three Hours Maximum : 80 Ma

Part A (Objective Type)

Answer all **twelve** *questions.*

1. At what points are function $f(x) = \dfrac{1}{(x+2)^2} + 4$ continuous ?

2. Define critical point of a function.

3. Suppose $\lim_{x \to c} f(x) = 5$ and $\lim_{x \to c} g(x) = -2$. Find $\lim_{x \to c} f(x)\, g(x)$.

4. Find the norm of the partition $[0, 1.2, 1.5, 2.3, 2.6, 3]$.

5. Find absolute minima of $y = x^2$ on $(0, 2]$.

6. Find the interval in which $y = x^3$ is concave up.

7. $\dfrac{d}{dx} \int_a^x f(t)\, dt =$ ————————.

8. Find dy if $y = x^5 + 37x$.

9. Define average value of a function f on $[a, b]$.

10. Find $\lim_{x \to -\infty} \dfrac{\pi\sqrt{3}}{x^2}$.

11. Define horizontal asymptote of the graph of a function.

12. Find $\lim_{x \to 2} \dfrac{3 - x}{3 + x}$.

(12 × 1 = 12 ma

Turn c

Part B (Short Answer Type)

Answer any nine questions.

13. If $2 - x^2 \le g(x) \le 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

14. If $\lim_{x \to 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \to 4} f(x)$.

15. Find the derivative of $y = \sqrt{x}$ for $x > 0$. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

16. Area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4} D^2$. How fast is the area changing with respect to the diameter when the diameter is 10 m ?

17. Find absolute extreme values of $g(t) = 8t - t^4$ on $[-2, 1]$.

18. Show that $\lim_{x \to -\infty} \frac{1}{x} = 0$.

19. The radius r of a circle increases from $r_0 = 10\ m$ to 10.1 m. Estimate the increase in the circle's area A by calculating dA. Compare this with true change ΔA.

20. Find a lower bound for the value of $\int_0^1 \cos x \, dx$ using the inequality $\cos x \ge 1 - x^2/2$.

21. Use Max-Min inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1 + x^2} dx$.

22. Find the area of the region between $y = 4 - x^2, 0 \le x \le 3$ and the x-axis.

23. Find the function with derivative $f'(x) = 2x - 1$ passing through the point $P(0, 0)$.

24. Find $\frac{d}{dx} \int_0^{t^4} \sqrt{u} \, du$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Find the slope of the curve $y = 1/x$ at $x = a$. Where does the slope equal $-1/4$? What happens to the tangent to the curve at the point $(a, 1/a)$ as a changes ?

26. Show that functions with zero derivatives are constant.

27. Find the asymptotes of the graph of $f(x) = \frac{-8}{x^2 - 4}$.

28. Find $\lim_{x \to 0} + \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.

29. Show that functions with the same derivative differ by a constant.

30. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

31. Express the solution of the initial value problem $\frac{ds}{dt} = f(t), s(t_0) = s_0$ in terms of integral.

32. Show that if f is continuous on $[a, b], a \ne b$ and if $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.

33. Show that if f has a derivative at $x = a$ then f is continuous at a.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \le x \le 3$ is increasing and decreasing. What are the critical points ? When does the function assume extreme values and what are these values ?

35. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}y^3, x = 0, y = -1, y = 1$ about x-axis.

36. Let $f(x) = \begin{cases} 3 - x, & x < 2; \\ 2, & x = 2;. \\ \frac{x}{2} + 1, & x > 2 \end{cases}$

(a) Find $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to 2^-} f(x)$. and $f(2)$.

(b) Does $\lim_{x \to 2} f(x)$ exist ? If so, what is it ? If not, why not?

(c) Find $\lim_{x \to -2^+} f(x)$ and $\lim_{x \to -2^-} f(x)$.

(d) Does $\lim_{x \to -2} f(x)$ exist ? If so, what is it ? If not, why not ?

(2 × 10 = 20 marks)