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THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(U.G.—CCSS)

(Complementary Course)

MM 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

I. Objective Type Questions : Answer *all* questions :

Each question of weightage ¼.

1 Check for exactness :

$$\left(x^3 + 3xy^2\right) dx + \left(3x^2 y + y^3\right) dy = 0.$$

2 Curves that intersect a given curve at right angels are called —————.

3 Solve $y' = \log x$.

4 When is a square matrix A said to be non-singular ?

5 Find the characteristic roots of $\begin{bmatrix} 1 & 3 \\ 10 & 2 \end{bmatrix}$.

6 If '2' is an eigenvalue of a square matrix A, give one root of A^T.

7 Define a solenoidal vector.

8 When are two vectors $\bar{a}$ and $\bar{b}$ said to be orthogonal ?

9 What is the divergence of $\bar{a} = \left[x^2, y^2, z^2\right]$?

10 Find a unit normal vector to the surface $S : x^2 + y^2 + z^2 = a^2$.

11 Give the parametric representation of the plane $3x + 2y + z = 6$.

12 If $\bar{F} = \text{grad } f$, $f = x^2 + y^2 + 2z^2$, find $\int_C \bar{F} \text{ od } \bar{r}$ where C has initial point $A(0, 0, 0)$ and

terminal point $B : (2, 2, 2)$.

(12 × ¼ = 3 weightage)

II. Short Answer Type Questions : Answer *all* questions.

Each question of weightage 1.

13 Find the rank of the following matrix

$$A = \begin{bmatrix} 4 & 7 & 6 \\ 8 & 14 & 12 \\ 1 & 1 & 1 \end{bmatrix}.$$

14 State Cayley Hamilton Theorem.

15 Solve $\left(1 + x^2\right) \dfrac{dy}{dx} = 1 + y^2$, $y(0) = 1$.

16 Find an integrating factor for $2x \tan y \, dx + \sec^2 y \, dy = 0$.

17 Find the work done by $\vec{p} = [2, 6, 6]$ if it displaces a body from $A:(3, 4, 0)$ to $B:(5, 8, 0)$.

18 Prove that $\text{div}[\text{grad } f] = \nabla^2 f$.

19 Find the tangential and normal accelerations of $\vec{r}(t) = [b \cos t, b \sin t, c]$.

20 Check for path independence :

$$\sinh xz \,(z\,dx - xz\,dz).$$

21 Use Green's Theorem to evaluate $\oint\limits_{C} \vec{F} \text{ od } \vec{r}$, where $\vec{F} = \text{grad}\,(\sin x \cos y)$ and C is the ellipse

$25x^2 + 9y^2 = 225$.

(9 × 1 = 9 weightage)

III. Short Essay or Paragraph Questions : Answer any *five* questions :

Each question of weightage 2.

22 Solve $2xyy' = y^2 - x^2$.

23 Find the rank by reducing to normal form :

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}.$$

24 Find the eigenvalues of $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

25 Find the directional derivative of $f = \left(x^2 + y^2 + z^2\right)^{-1/2}$ at $(3, 0, 4)$ along $\vec{a} = [1, 1, 1]$.

26 Find the length of the hypocycloid $\vec{r}(t) = a \cos^3 t \, \hat{i} + a \sin^3 t \, \hat{j}$.

27 Test for exactness and hence evaluate :

$$\int_{\left(3, \, 3/2\right)}^{\left(4, \, 1/2\right)} 2x \sin \pi y \, dx + \pi x^2 \cos \pi y \, dy.$$

28 Evaluate using the Gauss Divergence Theorem : $\iint_S \bar{F} \text{ on } dA$, $\bar{F} = \left[x^3, y^3, z^3\right]$ and S is the

surface of the sphere $x^2 + y^2 + z^2 = 9$.

$(5 \times 2 = 10 \text{ weightage})$

IV. Essay Questions : Answer any *two* questions.
Each question of weightage 4.

29 Find the Orthogonal Trajectories of $y = c\sqrt{x}$.

30 Use Cayley Hamilton Theorem to find A^3 and A^4 if $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

31 Verify Stokes' Theorem for $\bar{F} = [y, z, x]$ and S is the portion of the paraboloid

$z = 1 - \left(x^2 + y^2\right)$, $z \geq 0$.

$(2 \times 4 = 8 \text{ weightage})$