D 14144 (Pages : 3) Name..................................

Reg. No..................................

THIRD SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, NOVEMBER 2016

(UG–CCSS)

Complementary Course

MM 3C 03—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage of ¼.

1. An additional solution of a differential equation that cannot be obtained from the general solution is called ———.

2. Solution of the initial value problem $y' = \frac{-y}{x}$; $y(1) = 1$ is :

   (a) $x^2 + y^2 = 1$.   (b) $xy = 1$.

   (c) $x = e^y$.   (d) $y = e^x$.

3. A family of curves such that each member of this family cuts all the curves of another family at right angles is called ———.

4. A is an $n \times n$ matrix and the rank is r. Then A is non-singular if :

   (a) $n = r$.   (b) $n < r$.

   (c) $n > r$.   (d) None of these.

5. A system of n non-homogeneous equations in n unknown has a unique solution provided ———.

   (a) $|A| = 0$.   (b) $|A| \neq 0$.

   (c) $|A| > 0$.   (d) None of these.

6. The characteristic equation of a unit matrix of order 3 is ———.

7. State Cayley-Hamilton theorem.

8. The work done by a force $\overline{p} = [2, 6, 6]$ acting on a body, if the body is displaced from A (3, 4, 0) to B (5, 8, 0) is ———.

9. The parametric representation of the straight line through A (3, 1, 5) and in the direction of $\overline{b} = [4, 7, -1]$ is ———.

Turn over

10. Given $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. Then the value of div $\bar{r}$ is ———.

11. State Green's theorem in the plane.

12. $\overline{F}$ is a field defined on D and $\overline{F} = \nabla f$, for some scalar function f on D, then f is called ———.

(12 × ¼ = 3 weightage)

Part B

Answer **all** *nine questions.*
Each question carries 1 *weightage.*

13. Show that $\left(x^3 + 3xy^2\right)dx + \left(3x^2y + y^3\right)dy = 0$ is exact and hence solve.

14. Find the orthogonal trajectory of $y = ce^{-x}$.

15. Define Elementary transformations on a matrix.

16. Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$.

17. Find $\bar{p} = [0, 0, p_3]$ such that the resultant of p ; $\bar{q} = [4, 0, 8]$; $\bar{u} = [2, -2, 6]$ and $\bar{v} = [-1, 1, 3]$ is parallel to the xy-plane.

18. Define linear independence of three vectors. Also verity that [4, 2, 9] ; [3, 2, 1] ; [– 4, 6, 9] are linearly independent.

19. Find the length of $\bar{r}(t) = t\bar{i} + \cosh t\,\bar{j}$ from $t = 0$ to $t = 1$.

20. Evaluate $\int_C \overline{F} \cdot d\bar{r}$ where $\overline{F} = [-y, -xy]$ and C is the arc of the circle represented by $\bar{r}(t) = [\cos t, \sin t]$; $0 \leq t \leq \frac{\pi}{2}$.

21. Using Green's theorem evaluate $\oint_C \overline{F} o d\bar{r}$ where $\overline{F} = \left[3y^2, x - y^4\right]$ and C is the square with vertices (1, 1), (– 1, 1) (– 1, – 1) and (1, – 1).

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Find the integrating factor and hence solve :

$$2\cosh x \cos y \, dx = \sinh x \sin y \, dz.$$

23. Solve $y' + 2y = y^2$.

24. Verity Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

25. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$.

26. Find $\nabla^2 f$ using the formula $\nabla^2 f = \text{div}(\text{grad } f)$ and verify by direct differentiation for $f = \frac{xy}{z}$.

27. Verify $\text{div}\left(\text{curl } \bar{u}\right) = 0$ if $\bar{u} = \frac{1}{2}\left(x^2 + y^2 + z^2\right)\left(\bar{i} + \bar{j} + \bar{k}\right)$.

28. Write the formula for area of a plane region as a line integral over the boundary. Using this evaluate the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(5 × 2 = 10 weightage)

Part D

Answer any **two** *questions.*
Each question carries 4 *weightage.*

29. (a) Show that $\bar{F} = \left(e^x \cos y + yz\right)\bar{i} + \left(xz - e^x \sin y\right)\bar{j} + (xy + z)\bar{k}$ is conservative and find the scalar potential for it.

    (b) Verify Green's theorem for $\bar{F} = (x - y)\,\bar{i} + x\bar{j}$ where R is the region bounded by the unit circle $\bar{r}(t) = \cos t\,\bar{i} + \sin t\,\bar{j}\,; 0 \le t \le 2\pi$.

30. (a) Find the characteristic roots and vectors of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

    (b) Using Cayley-Hamilton theorem find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$.

31. (a) Solve $y' = (x + y - z)^2$.

    (b) Solve $y' + y \sin x = e^{\cos x}$.

(2 × 4 = 8 weightage)