C 3992 (Pages : 3) Name.....

Reg. No.....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2016

(CUCBCSS—UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Objective type.
Answer all **twelve** *questions.*

1. What do you mean by a non linear differential equation ?
2. Write the solution for an homogeneous differential equation with $2 \pm 3i$ as a double root for its auxillary equation.
3. State the Existence and Uniqueness theorem for initial value problem.
4. What is the Shifting property of Laplace transform ?
5. Find $L^{-1}\left(\frac{1}{(s+1)^2}\right)$.
6. What is $L\left(t^{-1/2}\right)$?
7. How do you define a sawtooth wave function ?
8. Define and give an example of an odd function.
9. Write the Euler's formula for a fourier series of a periodic function.
10. Write the formula for Runge Kutta method.
11. Does the inital value problem $xy' = 4y$, $y(0) = 1$ has solution ? Give reason.
12. Give a formula for an error for Simpson's rule.

(12 × 1 = 12 marks)

Part B

Short answer type.
Answer any **nine** *questions.*

13. Find the particular integral for $y' + 4y = 8x^2$.
14. Find a basis for the solution of the differential equation $y' - y = 0$.
15. Find $W\left[e^{\lambda x}, xe^{\lambda x}\right]$.

Turn over

16. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at} F(t)$.

17. Show that the Laplace transform is a linear operation.

18. Find L $(\sin^2 t)$.

19. Show that the function $f(x)$ = constant is a periodic function of period p for every positive p.

20. Find the fourier series of $f(x) = -1, -\pi < x < \pi$.

21. Show that $u = \cos 4t \sin 2x$ is a solution of the wave equation.

22. Apply Picard's iteration upto 4 steps to solve $y' = y$ and $y(0) = 1$.

23. Show that the initial value problem $y' = \sqrt{|y|}, y(0) = 0$ does not have a unique solution.

24. What do you mean by Lipschitz condition ?

(9 × 2 = 18 marks)

Part C

Short essay.
Answer any **six** *questions.*

25. Solve $x^2y'' + 7xy' + 13y = 0$.

26. Verify $y_p = 2x^2 - 6x + 7$ is a solution for $y'' + 3y' + 2y = 4x^2$ and find a general solution.

27. Find the Laplace transform of $f(t) = \begin{cases} t, 0 < t < 4 \\ 5, t > 4 \end{cases}$.

28. Find $L^{-1}\left(\frac{3s+16}{s^2 - s - 6}\right)$.

29. Solve $u_y = 2xyu$.

30. Find the cosine series of $f(x) = x$, $0 < x < L$.

31. Solve the integral equation $y(t) = t + \int_0^t y(\tau) \sin(t - \tau)\, d\tau$.

32. Using Simpson's rule evaluate the integral $\int_1^2 x\, dx$ with $n = 4$ and hence find an upper bound for the error incurred.

33. Apply improved Euler method in 3 steps to solve $y' = y$, $y(0) = 1$ with $h = 0.1$. Also find the error occurred.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*

34. (a) Solve $x^2y'' - 4xy' + 6y = 21x^{-4}$.

(b) Solve the initial value problem $(D^2 + 4)\,y = -12 \sin 2x$, $y(0) = 1.8$, $y'(0) = 5$.

35. Solve the integral equation $y(t) = t + \int_0^t y(\tau) \sin(t - \tau)\, d\tau$.

36. Find the fourier series of $f(x) = \begin{cases} x^2, -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \frac{\pi^2}{4}, \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$.

(2 × 10 = 20 marks)