D 41988 **(Pages : 3)** Name..............................

Reg. No..........................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018

(CUCBCSS—UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 M

Part A (Objective Type)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What do you mean by fundamental period ?
2. Write the characteristic equation of the differential equation $2y'' + 10y' + 25y = 0$.
3. Write the 2 dimensional wave equation.
4. Give an example of a function which is neither odd nor even.
5. State the existence and uniqueness theorem for initial value problem.
6. Find the Laplace transform of $f'(t)$.
7. Find the Wronskian of $e^{\lambda x}$ and $xe^{\lambda x}$.
8. Find $L^{-1}\left(\dfrac{1}{s/a - 1}\right)$.
9. Find $L(t + e^t)$.
10. State the second shifting theorem of Laplace transform.
11. Write the error estimate for Simpson's rule.
12. Write the formula for Runge-Kutta method.

(12 × 1 = 12 m

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the Fourier cosine series of $f(x) = \pi - x, 0 < x < \pi$.
14. Show that $u = e^{-w^2c^2t}\sin wx$ is a solution of heat equation.
15. Apply Picard's iteration to solve $y' = y - x^2, y(0) = 1$. Also find y (0, 1) and y (0, 2).
16. By reducing to first order solve $y'' + (y')^2 = 0$.

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17. Find fundamental set of solutions of $2t^2y''+3ty'-y=0 \;\; t>0$. Given that $y_1(t)=t^{-1}$ is a solution.

18. Solve $y''+8y'+16y=0$.

19. Find the inverse transform of $\frac{4}{(s+1)(s+2)}$.

20. Solve the initial value problem $y''+y'-6y=1 \;\; y(0)=0, \;\; y'(0)=1$.

21. Find $L\left(te^{-t}\sin t\right)$.

22. Using convolution property find $L^{-1}\left[\frac{1}{s\left(s^2+4^2\right)}\right]$.

23. Use Trapezoidal rule with $n=4$ to estimate $\int_1^2 x^2dx$.

24. Solve the partial differential equation $u_{yy}+4u=0$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve $\left(x^2D^2+3xD+1\right)y=0$.

26. Find the general solution of the differential equation $y''-2y'+5y=5x^3-6x^2+6x$.

27. Find the Laplace transform of $e^{-2t}u(t-3)$.

28. Find $L^{-1}\left(\frac{se^{-2s}}{s^2+\pi^2}\right)$.

29. Find the Fourier series expansion of :

$$f(x)=\begin{cases}\sin x & 0<x<\pi \\ 0 & \pi<x<2\pi\end{cases}.$$

30. Solve $xy''-y'=(3+x)x^2e^x$.

31. Find the solution of $u_x+u_y=0$ by separating variables.

32. Find approximate solution of $y'+y=e^x \;\; y(0)=0$.

33. Obtain the half-range cosine series of $f(x)=x$ when $0<x<2$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the non-homogeneous equation : $y'' - y' - 2y = 10\cos x$.

35. Solve the initial value problem $y'' + 2y' + 2y = r(t)$

$$\text{where } r(t) = \begin{cases} 10\sin 2t & \text{if } 0 < t < \pi \\ 0 & \text{if } t > \pi \end{cases}$$

$$y(0) = 1 \quad y'(0) = -5.$$

Using Laplace transforms.

36. Find the Fourier series of the functions

$$f(x) = \begin{cases} x + x^2 & -\lambda < x < \pi \\ \pi^2 & x = \pm\pi \end{cases}$$

Deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots = \frac{\pi^2}{6}$.

(2 × 10 = 20 marks)