

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCBCSS-UG)

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

*Answer all the twelve questions.**Each question carries 1 mark.*

1. State the Existence and Uniqueness theorem for initial value problem.
2. Define and give an example of an even function.
3. What do you mean by a non-linear differential equation ?
4. Solve $y'' - y' - 2y = 0$.
5. Define a unit step function.
6. State the existence theorem for Laplace transforms.
7. Find $L^{-1}\left(\frac{a}{s^2 - a^2}\right)$.
8. Find $L\left(t^{-1/2}\right)$.
9. Define a rectangular wave.
10. Write the 2-dimensional Poisson equation.
11. Give a formula for an error for Simpson's rule.
12. Write the formula for Runge Kutta method.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

*Answer any nine questions.**Each question carries 2 marks.*

13. Find the particular integral for $y' + 4y = 8x^2$.
14. Find a basis for the solution of the differential equation $y'' + y = 0$.

Turn over

15. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at}F(t)$.
16. Solve $3y'' - 8y' - 3y = 0, y(-3) = 1, y(3) = \left(\frac{1}{e^2}\right)$.
17. Find $L(e^{-\alpha t} \cos \beta t)$.
18. If $f(x)$ is a periodic function of x of period p , show that $f(ax), a \neq 0$, is a periodic function of x of period $\frac{p}{a}$.
19. Find the Fourier series of $f(x) = x + |x|, -\pi < x < \pi$.
20. Show that $u = e^{-t} \sin x$ is a solution of heat equation.
21. Apply Picards iteration to solve $y' = y - x^2, y(0) = 1$ also find $y(0.1)$ and $y(0.2)$.
22. Evaluate $\int_{-3}^3 x^4 dx$ using Simpson's rule.
23. What do you mean by convolution ?
24. Evaluate $\int_0^6 \frac{1}{1+x} dx$ by Trapezoidal rule.

(9 × 2 = 18 marks)

Part C (Short Essays)

*Answer any six questions.
Each question carries 5 marks.*

25. Solve $(4x^2D^2 + 12xD + 3)y = 0$.
26. Find a general solution of the differential equation $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$.
27. Find the Laplace transform of $(t-1)^2 u(t-1)$.

28. Find $L^{-1}\left(\frac{4(e^{-2s} - 2e^{-5s})}{s}\right)$.

29. Solve $u_{xy} = u_x$.

30. Find the Fourier series of $f(x) = \begin{cases} -k, & \text{if } -\pi < x < 0; \\ k, & \text{if } 0 < x < \pi, \end{cases}$ and $f(x+2\pi) = f(x)$.

31. Given $y' = -y$, $y(0) = 1$. Find the value of y at $x = (0.01)(0.01)(0.04)$ by improved Euler method.

32. Find approximate solution to $y' + y = e^x$, $y(0) = 0$.

33. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Simpson's rule.

(6 × 5 = 30 marks)

Part D

*Answer any two questions.
Each question carries 10 marks.*

34. Solve $x^2 y'' - 2xy' + 2y = x^3 \sin x$.

35. Solve the integral equation $y(t) = t + \int_0^t y(\tau) \sin(t-\tau) d\tau$.

36. Find the Fourier series $f(x) = \begin{cases} x + x^2, & \text{if } -\pi < x < \pi; \\ \pi^2, & \text{if } x = \pm \pi. \end{cases}$

(2 × 10 = 20 marks)