

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION MAY 2016

(UG—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Unit I

Answer all questions.

1. $\frac{d}{dx}(\operatorname{csc} h^{-1} x)$ is equal to :

(a) $\frac{1}{|x| \sqrt{x^2 + 1}}$

(b) $\frac{-1}{|x| \sqrt{x^2 + 1}}$

(c) $\frac{1}{x \sqrt{1 - x^2}}$

(d) $\frac{-1}{x \sqrt{x^2 - 1}}$

2. Write $\sinh^{-1} x$ as a logarithmic function.

3. $\int \coth h 5x \, dx$ is equal to :

(a) $\log \sin h 5x + c$

(b) $\log \cosh 5x + c$

(c) $\frac{1}{5} \log \sin h 5x + c$

(d) $\frac{-1}{5} \log \cosh 5x + c$

4. Write the n^{th} term of the sequence $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$

5. Find $\lim_{x \rightarrow \infty} \left(\frac{\ln x}{x} \right)$.

6. Find the sum of the series $\sum_{n=1}^{\infty} \frac{5(-1)^n}{4^n}$.

7. Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{-n}{2n+5} \right)$.

Turn over

8. Define the conditional convergence of the series Σa_n .
9. Find the spherical co-ordinate equation of the sphere $x^2 + y^2 + (z - 1)^2 = 1$.
10. Find the domain and range of the function $w = \sin xy$.

11. Find $\lim_{(x, y) \rightarrow (0, 1)} \left(\frac{x - xy + 3}{x^2 y + 5xy - y^3} \right)$.

12. Find $\frac{\partial^2 w}{\partial x \partial y}$ if $w = xy + \frac{e^4}{y^2 + 1}$.

(12 $\times$ $\frac{1}{4}$ = 3 weightage)

Unit II

Answer any **nine** questions.

13. Differentiate $t^2 \tan h \frac{1}{t}$ w.r. to t .
14. Find $\int \csc h^2 (5 - x) dx$.
15. Find $\int_{-\infty}^{\infty} \frac{2x}{(x^2 + 1)^2} dx$.
16. Does $\sum_{n=1}^{\infty} \frac{\log n}{n^{3/2}}$ converge? Why?
17. For what value of x the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$ converges?
18. Find the Taylor series expansion of $f(x) = x^2 - 2x + 4$ at $x = 2$.
19. Find the polar equation of the circle $x^2 + (y - 3)^2 = 9$.
20. Find the centre and radius of the circle $r = 4 \sin \theta$.

21. Find $\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^2 - xy}{\sqrt{x} - \sqrt{y}} \right)$.
22. Find f_x, f_y, f_z if $f(x, y, z) = 1 + xy^2 - 2z^2$.
23. Define the linearization of $f(x, y)$ at (x_0, y_0) .
24. Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$.

(9 × 1 = 9 weightage)

Unit III*Answer any five questions.*

25. Evaluate $\int_0^1 \frac{2 dx}{\sqrt{3+4x^2}}$.
26. Show that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.
27. Using geometric series prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
28. Find the Maclaurin's series for $\cos x$.
29. Find the points of intersection of the pair of curves $r = 4 \cos \theta, r = 1 - \cos \theta$.
30. Find the tangent plane to the surface $z = x \cos y - y e^x$ at $(0, 0, 0)$.
31. Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly at $(1, 1)$.
32. Find the area of the region enclosed by $r = 2(1 + \cos \theta)$.

(5 × 2 = 10 weightage)

Turn over

Unit IV

Answer any two questions.

33. (a) State and prove the Leibnitz theorem for the alternating series $u_1 - u_2 + u_3 - u_4 + \dots$
- (b) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$.
34. Find the points of intersection of the curves $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.
35. Find the region enclosed by the cardioid $r = 2 \cos \theta + 1$.

(2 × 4 = 8 weightage)