

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018
(CUCBCSS-UG)

Complementary Course
MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

*Answer all twelve questions.
Each question carries 1 mark.*

1. Write an example for a sequence which has no upper bound.
2. Find the domain of the function $w = xy \ln z$.
3. Define the level surface of a function f .
4. State two path test for non-existence of limit.
5. If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$.
6. $\frac{d}{dx} \sinh x =$ _____.
7. Write $\tanh x$ in terms of exponential function.
8. Find $\lim_{n \rightarrow \infty} \sqrt[n]{n}$.
9. $\int \cosh 2x =$ _____.
10. Find $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 + 2y}{3x - 2}$.
11. Find $\frac{\partial}{\partial x} \sin 2xy$.
12. Define conditional convergence of a series.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

*Answer any nine questions.
Each question carries 2 marks.*

13. Investigate the convergence of $\int_0^{\infty} e^{-x^2} dx$.
14. Show that $\lim_{n \rightarrow \infty} k = k$, where k is a constant.

Turn over

15. Find $\lim_{n \rightarrow \infty} \frac{\cos n}{n}$.
16. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.
17. Show that the function $f(x, y) = \frac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.
18. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.
19. Use chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$. What is the derivative's value at $t = \pi/2$?
20. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.
21. Show that if u is a differentiable function of x whose values are greater than 1, then
- $$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}.$$
22. Graph the sets of points whose co-ordinates satisfies the condition $2\pi/3 \leq \theta \leq 5\pi/6$ (no restriction on r).
23. Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.
24. Find the directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** questions.
Each question carries 5 marks.

25. Compare $\int_1^\infty \frac{dx}{x^2}$ and $\int_1^\infty \frac{dx}{1+x^2}$ with limit comparison test.
26. Determine whether the series $\sum_{n=1}^\infty \frac{1}{n^2}$ convergent or divergent.
27. Find the linearization of the function $f(x, y) = x^2 + y^2 + 1$ at $(0, 0)$.
28. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2, x = r - s$ and $y = r + s$.
29. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.
30. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = e^{3x+4y} \cos 5z$.

31. Find the Maclaurin series for the function $f(x) = xe^x$.
32. Does series $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ convergent.
33. Find the surface area generated by revolving the curves $x = t + \sqrt{2}$, $y = \frac{t^2}{2} + \sqrt{2}t$, $-\sqrt{2} \leq t \leq \sqrt{2}$ about y -axis.

(6 × 5 = 30 marks)

Part D (Essay Type)

*Answer any two questions.
Each question carries 10 marks.*

34. Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.
35. Find the points of intersection of $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.
36. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the functions's local and absolute extrema values.

(2 × 10 = 20 marks)