C 5622 **(Pages : 3)** **Name..............................**

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2016

(CUCBCSS–UG)

Complementary Course

MAT 2C 02—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*

1. Define a smooth curve.

2. Find $\frac{d}{dt}\left(\tanh\sqrt{1+t^2}\right)$.

3. Find the first four terms of the sequence $a_n = (-1)^{n+1}\frac{1}{n}, n \geq 1$.

4. Give an example of a sequence which has no upper bound.

5. State Leibnitz's theorem for the convergence of sequence.

6. Show that $(2, 3\pi/4)$ lies on the curve $r = 2\sin 2\theta$.

7. Write the formula for finding the length of the curve in polar co-ordinates.

8. Find the Cartesian equation of the surface $z = r^2$.

9. Define level surface of f.

10. When we say that a function f is continuous ?

11. Which order of differentiation will calculate f_{xy} faster, x first or y first for $f(x,y) = x\sin y + e^y$.

12. Define $\cosh x$.

(12 × 1 = 12 marks)

Part B

Answer **all** *questions.*

13. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

14. Evaluate $\int_0^1 \frac{2dx}{\sqrt{3+4x^2}}$.

Turn over

15. Find the volume of the solid generated by revolving the region between the curve :

$y = \sqrt{x}, 0 \le x \le 4$ and the x-axis.

16. Find the n^{th} term of the sequence 1, – 4, 9, –16, 25, ...

17. Replace $r\cos\theta = -4$ to the equivalent Cartesian equation.

18. Graph the set of points whose polar co-ordinates satisfy $1 \le r \le 2, \theta = \pi/4$.

19. Find all polar co-ordinates pairs which label the same as $(2, -\pi/3)$.

20. Find $\lim_{(x, y) \to (1, 1)} \frac{x^2 - y^2}{x - y}$.

21. Find f_x if $f(x,y) = 2y/(y+\cos x)$.

22. Find the length of the curve $r = 1 - \cos^{\theta}$.

23. Find the directrix of the parabola $r = \frac{25}{10 + 10\cos\theta}$.

24. Show that $\lim_{(x, y) \to (3,-4)} \sqrt{x^2 + y^2} = 5$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*

25. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the x-axis to generate a solid. Find the volume of the solid by Washer Method by explaining steps in detail.

26. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

27. Show that $\frac{d}{dx}\left(\sinh^{-1}(u/a)\right) = \frac{1}{\sqrt{a^2 + u^2}} \frac{du}{dx}$.

28. Express 5.232323... as the ratio of two integers.

29. Check the convergence of Σa_n where $a_n = \begin{cases} n/2^n, & n \text{ odd} \\ 1/2^n, & n \text{ even} \end{cases}$.

30. Graph the curve $r = 1 - \cos^{\theta}$.

31. Find the equivalent Cartesian equation of $r = \dfrac{4}{2\cos\theta - \sin\theta}$.

32. Find $\displaystyle\lim_{(x,\,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

33. Find $\partial z / \partial x$ if $yz - \ln z = x + y$ defines z as a function of two independent variables x and y and the partial derivative exists.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*

34. Find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le 1/2$ about the x-axis.

35. Find the radius and interval of convergence of $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, \sum_{n=1}^{\infty} n!x^n$.

36. (a) Write the chain rule and draw the tree diagram for finding $\dfrac{\partial w}{\partial r}, \dfrac{\partial w}{\partial s}$ if $w = x^2 + y^2$ $x = r - s$, $y = r + s$.

(b) Using Implicit differentiation, find dy/dx if $x + \sin y - 2y = 0$.

(2 × 10 = 20 marks)