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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, February 2021

First Degree Programme under CBCSS

Mathematics

Core Course VIII

MM 1545 : ABSTRACT ALGEBRA I

(2015-2017 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define a binary operation on the set of all $n \times n$ matrices with real entries.

2. Give an example of an isomorphism from $(\mathbb{R}, +)$ to $(\mathbb{R}^+, .)$

3. Why $\langle Q, +)$ and $\langle \mathbb{Z}, +)$ are not isomorphic?

4. Check whether $\mathbb{Z}^+$ under addition is a group.

5. Give example of abelian and non-abelian groups of order 6.

6. Write the generators of $\mathbb{Z}_8$?

7. What are the subgroups of $\mathbb{Z}$?

8. Give an example of a finite group that is not cyclic.

P.T.O.

9. Write the identity permutation in S_n as a product of transpositions.

10. Find all cosets of the subgroup $4\mathbb{Z}$ of $\mathbb{Z}$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions from this section. Each question carries **2** marks.

11. Prove that in a binary structure $\langle S, *\rangle$ if there is an identity element, it is unique.

12. Determine whether the diagonal n × n matrices with no zeros on the diagonal is a subgroup of $GL(n, \mathbb{R})$.

13. Prove that every cyclic group is abelian.

14. Describe the group of symmetries of a square.

15. Prove that every permutation σ of a finite set is a product of disjoint cycles.

16. Find the number of elements in the set. $\{\sigma \in S_5 / \sigma(2) = 5\}$.

17. Find the order of the permutation (1 4 7).

18. Let G be a group and let $a \in G$. Prove that $\langle a^{-1}\rangle = \langle a\rangle$.

19. Prove that order of an element of a finite group divides the order of the group.

20. Give an example of a subgroup of a group G whose left cosets give a partition of G into just one cell.

21. Suppose that $|G| = pq$, where p and q are prime. Prove that every proper subgorup of G is cyclic.

22. Find the order of (8, 4, 10) in the group $\mathbb{Z}_{12} \times \mathbb{Z}_{60} \times \mathbb{Z}_{24}$

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

23. Let G be a group and let $a \in G$. Prove that $H = \{a^n \mid n \in \mathbb{Z}\}$ is a subgroup of G and is the smallest subgroup of G that contains a.

24. Show that if H and K are subgroups of an abelian group G, then $\{hk \mid h \in H \text{ and } k \in K\}$ is a subgroup of G.

25. Find all subgroups of $\mathbb{Z}_{18}$ and give their subgroup diagram.

26. Show that S_n is non-abelian for $n \geq 3$.

27. Let σ be a permutation of a set A. For $a, b \in A$ define $a \sim b$ iff $b = \sigma^n(a)$ for some $n \in \mathbb{Z}$. Show that $\sim$ is an equivalence relation.

28. Let H be the subgroup $\langle \mu_1 \rangle = \{\rho_0, \mu_1\}$ of S_3. Find the partition of S_3 into left cosets of H and the partition into right cosets of H.

29. Let H be a subgroup of a finite group G. Show that the order of H is a divisor of the order of G.

30. Let $\sigma = (1\ 2\ 5\ 4)(2\ 3)$ in S_5. Find the index of $\langle \sigma \rangle$ in S_5.

31. Show that no permutation can be expressed both as a product of even number of transpositions and as a product of an odd number of transpositions.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions from this part. Each question carries **15** marks.

32. Describe the Klein-4 group and $\mathbb{Z}_4$ and determine their structural differences. Also draw their subgroup diagrams.

33. Let G be a cyclic group with generator a. Prove that if the order of G is infinite, then G is isomorphic to $(\mathbb{Z}_n, +)$ and if G has finite order n, then G is isomorphic to $(\mathbb{Z}_n, +_n)$.

34. Prove that every group is isomorphic to a group of permutations.

35. Let $G_1, G_2, \ldots, G_n$ be groups. For $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ in $\prod_{i=1}^{n} G_i$ define $(a_1, a_2, \ldots, a_n)(b_1, b_2, \ldots, b_n)$ to be the element $(a_1, b_1, a_2, b_2, \ldots, a_n b_n)$. Prove that $\prod_{i=1}^{n} G_i$ is a group.

(2 × 15 = 30 Marks)
