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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme under CBCSS

Mathematics

Core Course – 2

MM 1341 : ALGEBRA AND CALCULUS – 1

(2014 – 2017 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark. :

1. Define an unit element in a Ring.

2. A commutative skew field is ————.

3. A commutative ring with identity and ———— is an Integral domain.

4. Define Boolean Ring.

5. If $\vec{A} = 4\vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{B} = 3\vec{i} + 5\vec{j} + 2\vec{k}$ then find $\vec{A} \times \vec{B}$.

6. Evaluate $(\vec{i} + 2\vec{j} + 3\vec{k}) \cdot (\vec{i} + 3\vec{j} + 5\vec{k}) \times (\vec{i} + \vec{j} + 6\vec{k})$.

7. A vector V is called irrotational if ————.

P.T.O.

8. Angle between the lines whose direction cosines are $(l_1, m_1\ n_1)$ and (l_2, m_2, n_2) are ————————.

9. The equation of the straight line joining the points (2, 5, 8) and (–1, 6, 3) is ————————.

10. Find the equation of the sphere with centre (–1, 2, –3) and radius 3 units.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Define a Group. Is the set of integers under multiplication a group. Justify.

12. In an abelian group show that $(ab)^2 = a^2b^2$.

13. Prove that any unit in a Ring R cannot be a zero divisor.

14. Define characteristic of a ring and give one example.

15. Solve the linear congruence $5x \equiv 2 \pmod{26}$.

16. Determine a unit vector$\perp^{re}$ to the plane $\vec{A} = 2\vec{i} - 6\vec{j} - 3\vec{k}$ and $\vec{B} = 4\vec{i} + 3\vec{j} - \vec{k}$.

17. Find the area of the triangle having vertices at $P(1, 3, 2)$, $Q(2, -1, 1)$.

18. If $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ then find the work done in moving an object from (1, –2, 1) to (3, 1, 4).

19. Find the equation if the plane through (3, 4, 5) and parallel to $2x + 3y - z = 0$.

20. Find the distance between the parallel planes $2x - 2y - z + 3 = 0$, $4x - 4y + 2z + 5 = 0$.

21. Prove that the plane section of the sphere is a circle.

22. Show that the equation of a right circular cone whose vertex is O axis OZ and semi-verticle angle α is $x^2 + y^2 = z^2 \tan^2 \alpha$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks. :

23. Prove that a subgroup of a cyclic group is cyclic.

24. State and prove Fermat's theorem.

25. Prove that any field is an Integral domain.

26. Prove that $(Z_n, \oplus, \odot)$ is a Ring.

27. Find the unit tangent vector to any point on the curve $x = t^2 - t,\ y = 4t - 3,\ z = 2t^2 - 8t$.

28. Suppose $v = wx\ v$ where w is a constant then prove that $w = \frac{1}{2} curl\ v$.

29. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of a straight line then show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

30. Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$.

31. Find the equation of the Orthogonal projection of the line $\frac{x-2}{4} = \frac{y-1}{2} = \frac{z-4}{3}$ on to the plane $8x + 2y + 9z - 1 = 0$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks. :

32. Let R, R' be two Rings. Let $f : R \to R'$ be an Isomorphism then prove the following :
    (a) R is commutative $\Rightarrow R'$ is commutative.
    (b) R is a Ring with identity $\Rightarrow R'$ is a Ring with identity.
    (c) R is an Integral Domain $\Rightarrow R'$ is an Integral Domain
    (d) R is a field $\Rightarrow R'$ is a field.

33. (a) Define Kernel of a Ring homomorphism.

(b) State and prove fundamental theorem of Ring homomorphism.

34. State and prove Chinese remainder theorem.

35. (a) Prove that the system of Linear congruence

$ax + by \equiv r \pmod{n}$
$cx + dy \equiv s \pmod{n}$ has a unique solution modulo n whenever $\gcd(ad - bc, n) = 1$.

(b) Find the solution of

$7x + 3y \equiv 10 \pmod{16}$
$2x + 5y \equiv 9 \pmod{16}$

solution is $x \equiv 3 \pmod{16}$
$y \equiv 7 \pmod{16}$.

(2 × 15 = 30 Marks)