(Pages : 4) **J – 1188**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, March 2020

First Degree Programming under CBCSS

Mathematics

Core Course III

MM 1441 : ALGEBRA AND CALCULUS II

(2014 – 17 Admissions)

Time : 3 Hours Max. Marks : 80

UNIT I

Answer **all** questions from this unit. Each question carries **1** marks.

1. Find $(x^3+1)(2x^2+2)$ in $\mathbb{F}_3[x]$.

2. Find the remainder when $3x^2-x-1$ is divided by x^3-2.

3. For which values of k in Q, does $x-k$ divide $x^3-kx^2-2x+k+3$.

4. State whether the polynomials $x-1$ and $2x+3$ are associates in $\mathbb{Z}/5\mathbb{Z}[x]$.

5. Give an example of an irreducible polynomial of degree 2 in $\mathbb{R}[x]$.

6. Suppose that $z=x^2y, x=t^2, y=t^3$. Use the chain rule to find $\frac{dz}{dt}$.

P.T.O.

7. What is the natural domain of the function $f(x, y, z)=\sqrt{1-x^2-y^2-z^2}$.

8. Write f_x if $f(x,y)=2x^3y^2+2y+4x$.

9. Evaluate $\lim_{(x,y)\to(-1,2)} \frac{xy}{x^2+y^2}$.

10. Find $\int_0^3\int_1^2 (1+8xy)\,dy\,dx$.

(10 × 1 = 10 Marks)

UNIT II

Answer **any eight** questions from this unit. Each question carries **2** marks.

11. Using Fermat's theorem, find two different polynomials of degree 3 with coefficients in $\mathbb{Z}/3\mathbb{Z}$.

12. Let $R=\mathbb{Z}/4\mathbb{Z}=\{0, 1, 2, 3\}$. Find all the units of $R[x]$.

13. Find all roots of $f(x)=x^2-2x$ in $\mathbb{Z}/5\mathbb{Z}$.

14. Let $N(e)$ be the number of elements of U_p, the group of units of $\mathbb{Z}/p\mathbb{Z}$ which have order e. Then show that $\Sigma_{e/p-1}N(e)=p-1$.

15. Let $f(x)=x^2+bx+4$ in $\mathbb{R}[x]$. For what values of b, $f(x)$ is irredicible?

16. Decompose into partial fractions: $\frac{t+1}{(t-1)(t+2)}$.

17. Describe the level surfaces of $f(x, y, z)=z^2-x^2-y^2$.

18. Find $\frac{\partial z}{\partial x}$ if $z=x^4\sin(xy^3)$.

19. Show that when f is differentiable, a function of the form $z=f(xy)$ satisfies the equation $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=0$.

20. Show that the function $u(x,t)=\sin(x-ct)$ is a solution of $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$.

21. Find the parametric equation for the surface generated by revolving the curve $y=\frac{1}{x}$ about the x-axis where $1\leq x\leq 5$.

22. Evaluate the triple integral $\iiint_G 12xy^2z^3\,dV$ over the rectangular box G defined by the inequalities $-1\leq x\leq 2,\ 0\leq y\leq 3,\ 0\leq z\leq 2$.

(8 × 2 = 16 Marks)

UNIT III

Answer **any six** question from this unit. Each question carries **4** marks.

23. Solve the equation $x^3+6x=20$ by Cardano's method.

24. In $Q[x]$, find the g.c.d. of x^6-1 and x^4-1. Write the g.c.d. as in Bezout's identify.

25. State and prove Remainder Theorem. State roots theorem.

26. Prove that for any n, $\Sigma_{d/n}\phi(d)=n$.

27. Find the slope of the sphere $x^2+y^2+z^2=1$ in the y-direction at the points $\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$ and $\left(\frac{2}{3},\frac{1}{3},\frac{-2}{3}\right)$.

28. Suppose that $w=x^2+y^2-z^2$ and $x=\rho\sin\phi\cos\theta$, $y=\rho\sin\phi\sin\phi, z=\rho\cos\phi$. Find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$.

29. Use Lagrange Multipliers to determine the dimensions of a rectangular box open at the top, having a volume of 32 ft^3 and requiring the least amount of material for its construction.

30. Find an equation of the tangent plane to the parametric surface $x = uv, y = u, z = v^2$ at the point which correspond to $(u, v) = (2, -1)$.

31. Use a polar double integral to find the area enclosed by the three petaled rose $r = \sin 3\theta$.

(6 × 4 = 24 Marks)

UNIT IV

Answer **any two** question from this unit. Each question carries **15** marks.

32. (a) Solve the equation $x^3 - 7x + 6$.

(b) Find a solution of $y^4 - 5y + 6$.

33. (a) State fundamental theorem of Algebra.

(b) Prove that: no polynomial $f(x)$ in $R[x]$ of degree > 2 is irreducible in $R[x]$.

34. (a) Find the absolute maximum and minimum values of $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices $(0, 0)$, $(3, 0)$ and $(0,5)$.

(b) Locate all relative extrema and saddle points of $f(x, y) = 1 - x^2 - y^2$.

35. (a) Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the plane $z = 1$ and $x + z = 5$.

(b) Find the volume of the solid enclosed between the paraboloids $z = 5x^2 + 5y^2$ and $z = 6 - 7x^2 - y^2$.

(2 × 15 = 30 Marks)
