

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme Under CBCSS

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III — CALCULUS AND LINEAR ALGEBRA

(2018 admission)

Time : 3 Hours

Max. Marks : 80

PART – A

All the **ten** questions are compulsory. They carry **1** mark each .

1. Define an exact **first degree** first order ODE.
2. Write the general form of Bernoulli's equation.
3. Give an example for a linear first order ODE.
4. When is a vector field said to be conservative?
5. Write the continuity equation.
6. Give the integral form for divergence.
7. Give an example for a periodic function.
8. Define commutator of two matrices A and B.

9. Find AB if  $A = \begin{pmatrix} 3 & -4 \\ -4 & 2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 5 & 2 \\ -7 & 3 \end{pmatrix}$ .

10. Define linear function of vectors.

(10 × 1 = 10 Marks)

### PART – B

Answer **any eight** questions from among the questions 11 to 22. These questions carry **2** mark each.

11. Solve the ODE  $y' + 2 \sin 2\pi x = 0$  by integration.

12. State Stoke's Theorem.

13. State divergence theorem.

14. Find the period and frequency of the function  $s = \frac{1}{2} \cot(\pi t - 8)$ .

15. Give an example of a function which fails to satisfy the Dirichlet conditions but expandable in a Fourier series.

16. Solve  $\frac{dy}{dx} = x + xy$ .

17. Verify that  $y = ce^{-4x} + 0.35$  is a solution of the ODE  $y' + 4y = 1.4$ .

18. Give an expression for the angular momentum of a solid body rotating with angular velocity  $\omega$  about an axis through the origin.

19. Write  $e^x$  as the sum of an even function and an odd function.

20. State divergence theorem.

21. Find the direction of the line of intersection of the planes  $x - 2y + 3z = 4$  and  $2x + y - z = 5$ .
22. If  $\vec{F} = 2xz\hat{i} + 2yz^2\hat{j} + (x^2 + 2y^2z - 1)\hat{k}$ , find  $\nabla \times \vec{F}$ .

(8 × 2 = 16 Marks)

### PART – C

Answer **any six** questions from among the following questions 23 to 31. These questions carry **4** marks each.

23. Solve  $x \frac{dy}{dx} + y = xy^3$ .
24. Solve  $(x + 2y^3) \frac{dy}{dx} = y$ .
25. Show that the equation  $(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2)dy = 0$  is exact and solve it.
26. From Ampere's law, derive Maxwell's equation in the case where the currents are steady, ie.,  $\nabla \times \vec{B} = \mu_0 \vec{J} = 0$ .
27. Find the vector area of the surface of the Hemisphere  $x^2 + y^2 + z^2 = a^2, z \geq 0$  by evaluating the line integral  $\vec{S} = \frac{1}{2} \oint_C \vec{r} \times d\vec{r}$  around its perimeter.
28. Represent  $f(x) = \begin{cases} 1, & -1 < x < 1 \\ 0, & |x| > 1 \end{cases}$  as a Fourier integral.
29. Find the angle between the vectors  $\vec{A} = 3\hat{i} + 6\hat{j} + 9\hat{k}$  and  $\vec{B} = -2\hat{i} + 3\hat{j} + \hat{k}$ .

30. Find the rank of the matrix  $\begin{pmatrix} 1 & -1 & 2 & 3 \\ -2 & 2 & -1 & 0 \\ 4 & -4 & 5 & 6 \end{pmatrix}$ .
31. Find the dimension of the space spanned by the vectors  $\{(1, 0, 1, 5, -2), (0, 1, 0, 6, -3), (2, -1, 2, 4, 1), (3, 0, 3, 15, -6)\}$ .

(6 × 4 = 24 Marks)

### PART – D

Answer **any two** questions from among the following questions 32 to 35. These questions carry **15** marks each.

32. Use variation of parameters method to solve  $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ , subject to the boundary conditions  $y(0) = y(\pi/2) = 0$ .
33. Given the vector field  $\vec{a} = y\hat{i} - x\hat{j} + z\hat{k}$  verify Stoke's theorem for the hemispherical sphere  $x^2 + y^2 + z^2 = a^2$ ,  $z \geq 0$ .
34. Find the Fourier series for  $|x|$  in  $[-\pi, \pi]$  and deduce that  $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ .
35. Given the matrices

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 0 & -1 \\ 4 & -2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

- (a) Find  $A^{-1}$ ,  $B^{-1}$ ,  $B^{-1}AB$  and  $B^{-1}A^{-1}B$ .
- (b) Show that the matrices  $B^{-1}AB$  and  $B^{-1}A^{-1}B$  are inverses.

(2 × 15 = 30 Marks)