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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2020

First Degree Programme Under CBCSS

Complementary Course for Chemistry/Polymer Chemistry

MM 1231.2 : Mathematics II

CALCULUS WITH APPLICATIONS IN CHEMISTRY – II

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** the **ten** questions. **Each** question carries **1** mark.

1. Find $\frac{\partial f}{\partial y}$ if $f(x,y) = \sin(x/y)$.

2. Define the gradient of a scalar field.

3. Give an example for an alternating series.

4. What is the sum of the numbers from 1000 to 2000.

5. State D' Alembert's Ratio test.

6. Find the divergence of the vector field $\bar{a} = x^2y^2z^2\bar{i} + y^2z^2\bar{j} + x^2z^2\bar{k}$.

7. Evaluate $\int_0^3\int_1^2 (1 + 8xy)\,dy\,dx$.

P.T.O.

8. Define irrotational vector field.

9. Evaluate $1^3 + 2^3 + 3^3 + \ldots + 6^3$.

10. Evaluate $\int_0 \int_0 \int_0 (x^2 - y^2 + z^2)\, dz\, dy\, dx$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions from among questions 11 to 22. **Each** question carries **2** marks.

11. Find the total derivative of $f(x,y) = x^2 + 3xy$ with respect to x, given that $y = \sin^{-1} x$.

12. Verify whether $df = x^2 dy - (y^2 - xy)dx$ is an exact differential.

13. Sum the series $S = 2 + \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots$.

14. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+2)}$.

15. Determine whether the following series converges:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \ldots$$

16. Find the equations of the tangent plane and normal line to the surface of the sphere $\phi = x^2 + y^2 + z^2$ at the point (0,0, a).

17. Show that curl grad $\phi = \vec{0}$

18. Show that $\nabla \cdot (\nabla\phi \times \nabla\psi) = 0$ where ϕ and ψ, are scalar fields.

19. Evaluate the double integral $I = \iint_R x^2 y\, dx\, dy$ where R is the triangular area bounded by the lines x = 0, y = 0 and x + y = 1.

20. Find the Laplacian of $\varphi = x^2 + y^2 - 2z^2$.

21. Find the centre of mass of the solid hemisphere bounded by the surfaces $x^2 + y^2 + z^2 = a^2$ and the xy-plane, assuming that it has a uniform density ρ.

22. Find an expression for a volume element in spherical polar coordinates.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions from the questions 23 to 31. **Each** question carries **4** marks.

23. Transform the expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ into polar co-ordinates.

24. The temperature of a point *(x, y)* on a unit circle is given by $T(x, y) = 1 + xy$. Find the temperature of the two hottest points on the circle.

25. Determine the range of values of x for which the following power series converges: $P(x) = 1 + 2x + 4x^2 + 8x^3 + \ldots$

26. Expand *f(x)* = *cos x* as a Taylor series about $x = \pi / 3$.

27. Prove that the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

28. Find the volume of the tetrahedron bounded by the three coordinate surfaces $x = 0$, $y = 0$ and $z = 0$ and the plane $x / a + y / b + z / c = 1$.

29. Find and evaluate the maxima, minima and saddle point of the function $f(x, y) = xy(x^2 + y^2 - 1)$.

30. Find the moment of inertia of a uniform rectangular lamina of mass M with sides a and b about one of the sides of length b.

31. Find the Taylor expansion up to quadratic terms in $x - 2$ and $y - 3$, of $f(x, y) = y\,exp(xy)$ about the point $x = 2$, $y = 3$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions from the questions 32 to 35. Each question carries **15** marks.

32. Find the stationary points of $f(x, y, z) = x^3 - y^3 + z^3$ subject to the constraints. $g(x, y, z) = x^2 + y^2 + z^2 = 1$ and $h(x, y, z) = x - y - z = 0$.

33. Derive the Frenet-Serret formulae for space curves.

34. Determine whether the following series converges:

(a) $\sum_{n=1}^{\infty} \frac{1}{n! + 1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{25} + \ldots$

(b) $\sum_{n=1}^{\infty} \frac{1}{(n!)^2} = 1 + \frac{1}{2^2} + \frac{1}{6^2} + \ldots$

35. Evaluate the double integral $I = \int\int_R \left(a + \sqrt{x^2 + y^2}\right) dxdy$ where R is the region bounded by the circle $x^2 + y^2 = a^2$..

(2 × 15 = 30 Marks)