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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2020

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1231.1: MATHEMATICS II – CALCULUS WITH APPLICATIONS IN PHYSICS – II

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer the **first ten** questions are compulsory. They carry **1** mark each.

1. Find the complex conjugate of the complex number $z = (x+5i)^{(3y+2ix)}$.

2. Find the sum of the complex numbers $1+2i$, $-3+4i$ and $2-6i$.

3. Find the argument of the complex number $z = 1+\sqrt{3i}$.

4. Evaluate $\frac{d}{dx}\left(\sinh^{-1\left(\frac{3}{x}\right)}\right)$.

5. Let $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ Find $\frac{\partial f}{\partial x}$.

6. State Pappu's second theorem.

7. Find $\int_0^1\int_0^1\int_0^1 dxdydz$.

8. Find the Laplacian of the scalar field $\varphi = xy^2z^3$.

9. Define $\nabla .a$ in spherical polar coordinates, where a is a vector field.

10. The position vector of a particle at time in Cartesian coordinates is given by $r(t) = 2t^2 i + (3t - 2)j + (3t^2 - 1)k$. Find the velocity of the particle.

SECTION II

Answer **any eight** questions among the questions **11 to 22.** These questions carry **2** marks each.

11. Evaluate $\text{Im}\ (\cosh^2 z)$.

12. Solve the hyperbolic equation $\cosh x - 5\sinh x - 5 = 0$.

13. Evaluate $\int e^{2x} \sin 3x dx$.

14. Show that the differential $df = x^2 dy - (y^2 + xy)dx$ is not exact.

15. Find the total derivative of $f(x,y) = x^2 + 3xy$ with respect to x, given that $y - \sin^{-1} x$.

16. Find the Taylor expansion, up to quadratic terms in $x - 2$ and $y - 3$ of $f(x,y) = ye^{xy}$ about the point $x = 2,\ y = 3$.

17. Evaluate $\int_1^3 \int_2^4 (40 - 2xy)dydx$.

18. A semi-circular uniform lamina is freely suspended from one of its corners. Show that its straight edge makes an angle of 23.0° with the vertical.

19. Evaluate the double integral $\iint_R x^2 y\ dxdy$, where R is the triangular area bounded by the lines $x = 0,\ y = 0$ and $x + y = 1$.

20. Find div F and curl F or the vector field F $(x,y,z) = x^2 i - 2j + yzk$.

21. For the function $\varphi = x^2y + yz$ at the point $(1,2,-1)$, find its rate of change with distance in the direction $a = i + 2j + 3k$.

22. The position vector of a particle in plane polar coordinates is $r(t) = p(t)\hat{e}p$. Find p expressions for the velocity and acceleration of the particle in these coordinates.

SECTION III

Answer any **six** questions among the questions **23 to 31**. These questions carry **4** marks each.

23. By writing $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ and considering $e^{i\pi/12}$, evaluate $\cot\left(\frac{\pi}{12}\right)$.

24. Prove that $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \ -1 < x < 1$.

25. Prove that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$, where $z = e^{i\theta}$.

26. The function $f(x,y)$ satisfies the differential equation $y\frac{\partial f}{\partial x} + x\frac{\partial f}{\partial y} = 0$. By changing to new variables $u = x^2 - y^2$ and $v = 2xy$, show that f is a function of $x^2 - y^2$ only.

27. The temperature of a point (x,y) on a unit circle is given by $T(x,y) = 1 + xy$. Find the temperature of the two hottest points on the circle.

28. Identify the curved wedge bounded by the surface $y^2 = 4ax$, $x + z = a$ and $z = 0$ and hence calculate its volume V.

29. Find the Jacobin $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ of the transformation $u = xy, v = y, w = x + z$.

30. Prove that curl $a = \nabla \times (\nabla \times a) = \nabla(\nabla \cdot a) - \nabla^2 a$.

31. For the twisted space curve given parametrically by $x = au(3-u^2), y = 3au^2$, $z = au(3+u^2)$. Show that the radius of curvature at u is $3a(1+u^2)^2$.

SECTION IV

Answer any **two** questions among the question **32 to 35**. These questions carry **15** marks each.

32. (a) Use the Moivre's theorem with $n = 4$ to prove that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$, and deduce that $\cos\frac{\pi}{8} = \left(\frac{2+\sqrt{2}}{4}\right)^{\frac{1}{2}}$.

(b) By diffentiating $e^{(a+ib)x}$ and separating real and imaginary parts, find the derivatives of $e^{ax}\cos bx$ and $e^{ax}\sin bx$.

33. (a) Find the stationary values of f $(x,y) = 4x^2 + 4y^2 + x^4 - 6x^2 - y^2 + y^4$ and classify them as maxima, minima or saddle points.

(b) Suppose that $w = \sqrt{x^2 + y^2 + z^2}$, $x = \cos\theta$, $y = \sin\theta$, $z = \tan\theta$. Find $\frac{dw}{d\theta}$ when $\theta = \pi/4$.

34. (a) Evaluate $\int_{-\infty}^{\infty} e^{-x^2} dx$.

(b) A tetrahedron is bounded by the three coordinate surfaces and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{x} = 1$ and has density $\rho(x,y,z) = \rho_0\left(1 + \frac{x}{a}\right)$. Find the average value of the density.

35. (a) Express the vector field $a = yzi - yj + xz^2k$ in cylindrical polar coordinates, and hence calculate its divergence.

(b) Derive Frenet-Serret formulae.
