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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2019

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1431.1 – MATHEMATICS IV (COMPLEX ANALYSIS, FOURIER SERIES AND FOURIER TRANSFORMS)

(2014 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They carry **1** mark each.

1. If $z_1 = 6 + 3i, z_2 = -2 + 3i$, what is the imaginary part of $\frac{z_1}{z_2}$?.

2. Using D'Moivre's Theorem, express $\sin 4\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$.

3. Define analyticity of a complex function $f(z)$ in a domain D in the complex plane.

4. State Cauchy — Riemann equations.

5. State Morera's Theorem.

6. Obtain the singular points of $f(z) = Co\sec z$.

7. Find the residue of $f(z) = \frac{1}{(z^2+1)^3}$ at $z = -i$.

8. State Dirichlet conditions for the convergence of a Fourier series of a function $f(x)$ of period 2π.

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9. Write the standard form of Fourier Sine series and formulae for Fourier coefficients of the half range Sine series of a function $f(x)$ in $(0, \pi)$.

10. If $F(s)$ is the Fourier transform of $f(x)$ then what is the Fourier transform of $f(x-a)$ where a is any real number.

(10 × 1 = 10 Marks)

SECTION – II

Answer any eight questions from among the questions 11 to 22. These question carry two marks each.

11. Find all distinct fourth roots of -1.

12. Prove that the real and imaginary parts of an analytic function are harmonic.

13. Find the real and imaginary parts of $f(z) = 2z^3 - 3z$.

14. Evaluate $\int_C \text{Re}\, z^2 dz$ where C is the unit circle in the counter clockwise direction.

15. Develop the function $f(z) = \dfrac{1}{z+3i}$ in a Maclaurin's series and find the radius of convergence.

16. Determine the location and nature of singularities of $f(z) = z^2 - \dfrac{1}{z^2}$.

17. Find the residues of $f(z) = \dfrac{1}{1-e^z}$ at its singular points.

18. Expand $f(z) = \dfrac{1}{z^3 - z^4}$ as a Laurent's series that converges for $|z| > 1$.

19. State Cauchy's Integral formula and using it evaluate $\oint_C \dfrac{z+1}{z^2} dz$ where C is a unit circle.

20. Find the half range Cosiine series of $f(x) = x, 0 < x < \pi$.

21. Expand $f(x) = (x-1)^2, 0 < x < 1$ in a Fourier series of Sine terms only.

22. Prove that Fourier transform is a linear operator.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions from among the questions 23 to 31. These questions carry 4 marks each.

23. If $f(z)$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

24. What are the values of $\int_C \frac{e^z}{z^2+1} dz$ if C is a unit circle with centre at

(a) $z = i$ and

(b) $z = -i$?

25. Find the Laurent's series for the function $f(z) = \frac{z}{(z-1)(z-3)}$ in $0 < |z-1| < 2$.

26. Obtain the residues of $f(z) = \frac{z^2 - z + 2}{(z+3i)(z-3i)(z+i)(z-i)}$ at its poles.

27. State Cauchy's Residue Theorem. Use Cauchy's Residue Theorem to evaluate the integral of the function $f(z) = \frac{1}{1+z^2}$ around the circle $|z| = 2$, in the positive sense.

28. Show that $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$.

29. Obtain the Fourier series of the periodic function defined by

$f(x) = \begin{cases} -\pi \text{ if } -\pi < x < 0 \\ x \text{ if } 0 < x < \pi \end{cases}$. Deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\infty$.

30. Obtain the Fourier series of periodicity 2 for $f(x)=\begin{cases} x \text{ if } -1 < x \le 0 \\ x+2 \text{ if } 0 < x \le 1 \end{cases}$

31. Find the Fourier transform of $f(x)=\begin{cases} Cosx, |x| < \frac{\pi}{2} \\ 0, \ |x| > \frac{\pi}{2} \end{cases}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **Two** questions from among the questions 32 to 35. These questions carry 15 marks each.

32. (a) If a function $f(z)$ is analytic, show that is independent of $\bar{z}$.

(b) Show that the function $u(x,y)=x^4-6x^2y^2+y^4$ is harmonic and find the corresponding analytic function $f(z)$ in terms of **z**.

33. (a) Expand $\frac{1}{1+z^2}$ as a Laurent series about $z=i$.

(b) Evaluate $\int_C \frac{Sin\pi\, z^2 + Cos\pi\, z^2}{(z-1)^2(z-2)} dz$ where C is $|z|=3$.

34. Expand $f(x)=x\sin x$ as a cosine series in $0 < x < \pi$ and deduce $1+\frac{2}{1.3}+\frac{2}{3.5}+\frac{2}{5.7}+\ldots\infty$.

35. Find the Fourier transform of $f(x)=\begin{cases} 1-x^2, |x| \le 1 \\ 0, |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{x\, Cosx - Sinx}{x^3} Cos\frac{x}{2} dx$.

(15 × 2 = 30 Marks)
