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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, February 2018
First Degree Programme Under CBCSS
COMPLEMENTARY COURSE I FOR PHYSICS
MM 1131.1 : Mathematics I – Differentiation and Analytic Geometry
(2014 Admission Onwards)

Time : 3 Hours Max Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **1** mark **each**.

1. What is Brachistochrone problem ?
2. Write the natural domain of the function $f(x) = \sqrt{x^2 - 4x + 3}$.
3. Find $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$.
4. State the focus-directrix property of conics.
5. Write down the degree of the homogenous function $\cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$.
6. What do you mean by a horizontal asymptote to a rational function ?
7. Write the Divergence test for the series $\sum u_k$.
8. Find the polar coordinates of the point P whose rectangular coordinates are $(-2, 2\sqrt{3})$.
9. Find an equation of the parabola that is symmetric about y-axis, has its vertex at the origin and passes through the point (5, 2).
10. Use horizontal line test to prove that the function $f(x) = x^2$ does not have an inverse.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Draw the position versus time curve for a particle with coordinate s_0 at time $t = 0$ and moving with constant velocity v.

12. What can you say about the continuity of the function $f(x) = \sqrt{9 - x^2}$?

13. Prove that a function which is differentiable at $x = x_0$ is continuous at x_0.

14. Find the coordinates of all points on the graph of $y = 1 - x^2$, at which the tangent line passes through the point (2, 0).

15. Verify Rolle's theorem for $f(x) = \cos x$ in $\left[\pi/2, 3\pi/2\right]$.

16. Find fxy, if $f(x, y) = \frac{x - y}{x + y}$.

17. If 'f' is homogenous function of degree 'n' prove that $x\,\partial f/\partial x + y\,\partial f/\partial y = nf$.

18. Find dy and Δy for $y = x^2 - 2x + 1$.

19. Sketch the parabola $(y - 3)^2 = 6(x - 2)$.

20. Write the local linear approximation of $f(x) = \frac{1}{x}$ at $x_0 = 2$ & approximate $\frac{1}{2.05}$.

21. Find the equation of the ellipse whose ends of major axis are $(\pm 6, 0)$ and passes through (2, 3).

22. Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$, where $x = \sin u + \cos v$, $y = -\cos u + \sin v$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

24. At what points is the tangent line to the curve $y^2 = 2x^3$ perpendicular to the line $4x - 3y + 1 = 0$?

25. Expand $\log_e^x$ in ascending powers of x – 1 and hence evaluate $\log_e(1.1)$ correct to four decimal places.

26. Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{5^k}{k^2} x^k$.

27. Find the vertical asymptote to the curve $f(x) = \frac{1}{(x-3)^2}$.

28. Find the maximum and minimum values of the function f(x, y) = 3x + 4y on the circle $x^2 + y^2 = 1$.

29. State Kepler's first, second and third laws.

30. Prove that $\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right)$ and also find $\int \frac{dx}{\sqrt{1+9x^2}}$.

31. Describe the graph of the equation $x^2 - 5y^2 - 2x - 10y - 9 = 0$.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) On a sunny day a 50 – ft flagpole casts a shadow that changes with the angle of elevation of the sun. Find the rate at which the length of the shadow is changing with respect to θ. When θ = 45°.

    b) Use Lagrange multiplier to find the maximum and minimum values of $f(x, y) = 4x^3 + y^2$, subject to the constraint $2x^2 + y^2 = 1$.

    c) A base ball diamond is a square whose sides are 90 ft long. Suppose that a player running from second base to third base has speed of 30 ft/s at the instant when he is 20 ft from the third base. At what rate is the player's distance from home plate changing at that instant ?

33. a) Verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ if $u = x^y + \sin(xy)$.

b) If $u = \sin^{-1}(x^3 + y^3)$ ST $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$.

c) If $w = e^{xyz}$, $x = 3u + v$, $y = 3u - v$, $z = u^2 v$. Use appropriate chain rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

34. a) Prove that the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_0, y_0) on it is $yy_1 = 2a(x + x_1)$.

b) Find the absolute maximum and minimum values of $f = x^3 - 3x^2 + 4$ on the interval $(0, \infty)$.

c) ST the Maclaurin's series for e^x converges to e^x.

35. a) Sketch the graph of $r = \frac{2}{1 + 2\sin\theta}$ in polar coordinates.

b) Find the asymptote to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

c) Sketch the graph of the hyperbola $\frac{x^2}{4} - \frac{y^2}{q} = 1$ showing their vertices, foci and asymptotes.
