(Pages : 4) **H – 2075**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2019

First Degree programme under CBCSS

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2 : MATHEMATICS I — DIFFERENTIATION AND MATRICES

(2014–2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define Horizontal asymptote.

2. Find the average rate of change of $y = x^2+1$ with respect to x over the interval [4, 6].

3. State Mean Value Theorem.

4. Find the derivative of f^{-1} where $f(x) = 5x^3 + x, -7$.

5. State the conditions for the existence of Maclaurin series representation of any function f(x) and write the formula for the Maclaurin series expansion of f(x).

6. State Euler's theorem for homogeneous functions.

7. Find the slope of the surface $f(x,y) = \sqrt{3x+2y}$ in the x - direction at the point (4, 2).

8. Define rank of a matrix.

9. Obtain the eigen values of $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$

10. Explain the method of diagonalization of a symmetric matrix.

SECTION – II

Answer **any 8** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Show that $y=x^3+3x+1$ satisfies $Y'''+xy''-2y'=0$.

12. Locate the relative extrema of the function $f(x)=x(x-1)^2$

13. Verify Rolle's theorem for the function $f(x)=x^2-6x+8$ over the interval [2,4].

14. The hypotenuse of a right triangle is known to be 10 units exactly and one of the acute angles is measured to be 30° with a possible error of $\pm 1^\circ$. Use the differentials to estimate the percentage error in the side opposite to the measured angle.

15. Obtain the interval of convergence of the power series $x-\frac{x^2}{2^2}+\frac{x^3}{3^2}-\frac{x^4}{4^2}+\ldots\infty$

16. Find the slope of the surface $f(x,y)=x^2y+5y^3$ in the x-direction at (1,-2).

17. Given that $x^3+y^2x-3=0$, find $\frac{dy}{dx}$ by implicit differentiation.

18. Verify Euler's Theorem for $f(x,y)=3x^2+y^2$

19. Locate all the critical points of $f(x, y) = 4xy-x^4-y^4$.

20. Reduce the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$ to its Row Echlon form.

21. Check for consistency using the concept of rank and solve $5x - 3y = 37, -2x + 7y = -38$.

22. If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of a matrix A of order n, then prove that $k\lambda_1, k\lambda_2, \ldots, k\lambda_n$, are the eigen values of the matrix kA where k is a nonzero constant.

SECTION – III

Answer **any 6** questions from among the questions **23** to 31. These questions carry **4** marks each.

23. Find $\frac{dy}{dx}$ at x=1 where $y = \frac{2x-1}{x+3}$

24. The position function of a particle moving along a coordinate line is given as $s(t) = t^3 - 6t^2, t \geq 0$.. Find the position, velocity, speed and acceleration at time t = 1.

25. Find the absolute maximum and minimum values of the function $f(x) = 4x^2 - 4x + 1$ $on [0,1]$ and state where these values occur.

26. Obtain Taylor series expansion of cos x in powers of $(x - \pi/4)$ up to three nonzero terms.

27. Show that the function $u(x,t) = \sin(x - ct)$ is a solution of the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

28. Obtain the Jacobian of transformation from Cartesian coordinates to Cylindrical polar coordinates.

29. For what values of a and b, the system of equations x+ y + z = 6, x + 2y + 3z = 10 and x + 2y + az = b possess infinitely many solutions? Obtain the solutions in this case.

30. Prove that the eigen values of a real symmetric matrix are real.

31. Diagonalize the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

SECTION – IV

Answer **any 2** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) Find a nonzero value for the constant k that makes $f(x)\begin{cases} \dfrac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2, & x \geq 0 \end{cases}$ continuous at $x = 0$.

(b) Find the height and radius of the cone of slant height L whose volume is as large as possible.

33. (a) Let $f(x) = \sqrt{4x - 3}$ and let c be a number that satisfies the Mean value theorem on [1 3]. What is c?

(b) If $f(x, y, z) = x + 2y + z^2,\ x = \dfrac{r}{s}, y = r^2 + in\, s, z = 2r$, find the partial derivatives $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$.

34. (a) Obtain the Maclaurin series expansion of $\tan^{-1} x$ by using the technique of term by term integration of the Power series.

(b) Find the maximum and minimum values of $f(x, y, z) = xyz$ on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$, using Lagrange's multiplier method.

35. (a) Examine the following system of equations for consistency and solve: $x + 2y + z = 2,\ 3x + y - 2z = 1,\ 4x - 3y - z = 3$.

(b) Find a basis of eigen vectors and diagonalize the matrix $\begin{bmatrix} 16 & 0 & 0 \\ 48 & -8 & 0 \\ 84 & -24 & 4 \end{bmatrix}$.
