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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2020

First Degree Programme Under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2014 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. $27x \equiv 1 (\text{mod } 31)$ has a solution. Justify the statement.

2. Write down the elements of $[7]_{12}$.

3. Define a complete set of representatives for $\mathbb{Z}/m\mathbb{Z}$.

4. Define a 2 pseudo prime.

5. Find the absolute minimum of $f(x) = 3x^4 + 4x^3$.

6. State the fundamental Theorem of calculus.

7. Give the arc length formula for parameterized curves.

P.T.O.

8. Evaluate $\lim_{x \to 0} \frac{1-\cos x}{x^2}$.

9. Show that for $p > 1$, $\int_1^\infty \frac{dx}{x^p} = \frac{1}{p-1}$.

10. Find the rectangular co-ordinates of the point whose polar coordinates are $\left(6, \frac{2\pi}{3}\right)$.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Prove that, if $a \equiv b (\text{mod } m)$ and d/m, then $a \equiv b (\text{mod } d)$.

12. Prove that, $[a]_m = [b]_m$ if and only if $a \equiv b (\text{mod } m)$.

13. Find the order of [4] in $\mathbb{Z}/7\mathbb{Z}$.

14. Show that Fermat's theorem is special case of Euler's Theorem.

15. Find the intervals on which $f(x) = x^3 - 4x + 3$ is increasing and is decreasing.

16. Find the point of inflection of $f(x) = \sin x$ on $[0, 2\pi]$.

17. State the second derivative test for twice differentiable functions.

18. State the theorem of differentiability of inverse functions.

19. Evaluate $\int \frac{dx}{\sqrt{4x^2 - 9}}$, $x > \frac{3}{2}$.

20. Find $\int_0^\infty (1-x)e^{-x} dx$.

21. Find the area enclosed by the cardioid $r = 1 - \cos\theta$.

22. Express $r = 2 + \cos\frac{5\theta}{2}$, parametrically.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Show that $2^{560} \equiv 1 \pmod{561}$.

24. Find $12^{100} \pmod{34}$.

25. Show that if *e* is the order of a modulo *m*, and $a^f \equiv 1 \pmod{m}$, there *e* divides *f*.

26. Evaluate $\int t^4 \sqrt[3]{3 - 5t^5}\, dt$ and $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$.

27. Find the area of the region enclosed by curves, $x = y^2$ and $y = x - 2$ by integrating with respect to *x*.

28. Show that $\frac{d}{dx}(\log_b x) = \frac{1}{x}\log_b e$.

29. Show that $\lim_{x \to 0}(1 + x)^{\frac{1}{x}} = e$.

30. Find $\int_0^1 \tan^{-1}x\, dx$.

31. Find the area enclosed by the region inside $r = 4 + 4\cos\theta$ and outside the circle $r = 6$.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Write the round by round match schedule with 8 players.

(b) State and prove Fermat's theorem in terms of congruence classes.

33. (a) Briefly explain the encoding and decoding procedures in RSA codes.

(b) Find the average value of $f(x) = \frac{\cos \frac{\pi}{x}}{x^2}$ over [1, 3].

34. (a) Find the volume of the right pyramid whose altitude is h and whose base is a square with sides of length a.

(b) Find the area of the surface obtained by revolving the region in the first quadrant bounded by $y = x^3$ between $x = 0$ and $x = 1$ about the x axis.

35. (a) Evaluate $\int \frac{2x+4}{x^3 - 2x^2} dx$.

(b) Sketch the graph of $r = \frac{6}{2 + \cos\theta}$, in polar co-ordinates.

(2 × 15 = 30 Marks)