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**Reg. No. : .............................**

**Name : ..................................**

## Sixth Semester B.Sc. Degree Examination, March 2021

## First Degree Programme under CBCSS

## Mathematics

## MM 1661.1 : GRAPH THEORY

## (Elective)

## (2015 – 17 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

**All** the first **ten** questions are compulsory. These questions carry **1** mark each.

1. Define Regular graph.

2. Define Parallel edges.

3. Define Spanning sub-graph.

4. Define null graph.

5. Give an example for an Euler graph that is a complete graph.

6. Define length of a path.

7. Distance between two vertices in a complete graph with more than one vertex is ____________.

P.T.O.

8. Define radius of a tree.

9. Define eccentricity of a vertex in a tree.

10. Define planar graph.

**(10 × 1 = 10 Marks)**

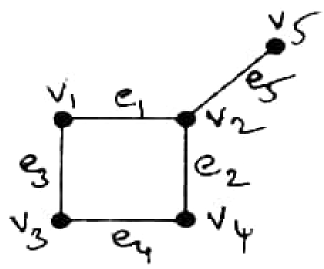
SECTION – II

Answer **any eight** questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Draw two non-isomorphic simple graphs on 4 vertices.

12. Prove that the sum of the degrees of all vertices in a graph $G$ is twice the number of edges in $G$.

13. Find the incidence matrix of the following graph




14. Distinguish between symmetric digraph and asymmetric digraph.

15. Distinguish between isolated vertex and pendent vertex.

16. Distinguish between connected and disconnected graphs.

17. Distinguish between edge disjoint and vertex disjoint sub graphs.

18. Prove that a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

19. Prove that there is one and only one path between every pair of vertices in a tree.

20. Prove that any simple planar graph can be embedded in a plane such that every edge is drawn as a straight line segment.

21. Distinguish between Range and Nullity.

22. Explain Four Colour Theorem.

**(8 × 2 = 16 Marks)**

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry **4** marks each.

23. Does there exists a simple graph on 5 vertices and 11 edges. Justify your answer.

24. Define self-complementary graph with an example.

25. Explain Decanting problem and its graph model.

26. Prove that a graph $G$ is disconnected if and only if its vertex set can be partitioned into two nonempty, disjoint subsets $V_1$ and $V_2$ such that there exists no edge in $G$ whose one end vertex is in subset $V_1$ and the other in subset $V_2$.

27. Explain Chinese Postman Problem.

28. Prove that in a connected graph $G$ with exactly $2k$ odd vertices, there exists $k$ edge-disjoint subgraphs such that they together contain all edges of $G$ and that each is a unicursal graph.

29. Prove that every tree has either one or two centers.

30. Prove that a graph $G$ with $n$ vertices. $n - 1$ edges, and no circuits is connected.

31. Prove that every connected graph has at least one spanning tree.

**(6 × 4 = 24 Marks)**

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) Distinguish between complete graph and complete bipartite graph. Give an example of a graph that is complete as well as complete bipartite.

    (b) Explain the multicolour cubes puzzle and its solution.

33. (a) Prove that a connected graph $G$ is Euler if and only if all vertices of $G$ are of even degree.

    (b) Prove that the distance between vertices of a connected graph is a metric.

34. (a) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.

    (b) Prove that $K_5$ and $K_{3,3}$ are not planar.

35. (a) Prove that a connected graph on $n$ vertices is a tree if and only if it has $n-1$ edges.

    (b) Define Konigberg Bridge Problem. Draw a graph representing the problem.

**(2 × 15 = 30 Marks)**