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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2020

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1231.1 — MATHEMATICS II – INTEGRATION AND VECTORS

(2014-2017 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They Carry **1** mark each.

1. Evaluate $\int \frac{\cos x}{\sin^2 x} dx$.

2. Suppose that a particle moves on a coordinate line so that its velocity at time t is $v(t) = \sin t \ m/s$. Find the distance traveled by the particle during the time interval $0 \le t \le \frac{\pi}{2}$.

3. Define average value of a function.

4. Find the area under the curve $y = \frac{1}{(3x+1)^2}$ over the interval [0, 1].

5. Write the formula for finding the arc length of the smooth curve $y = f(x)$ over $[a, b]$.

6. Find the domain of the vector valued function $\overrightarrow{r(t)} = \cos \pi t\hat{i} - \ln t\hat{j} + \sqrt{t-2}\hat{k}$.

7. Find the unit tangent vector to the graph of $\overrightarrow{r(t)} = t^2\hat{i} + t^3\hat{j}$ at the point where $t = 2$.

8. Define the curl of the vector field.

9. Sketch the vector field $\vec{F}(x, y) = \dfrac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$ by drawing some representative of non-intersecting vectors.

10. State the Fundamental Theorem of Work Integrals.

SECTION – II

Answer any **eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Suppose that a point moves along a curve $y = f(x)$ in the xy – plane in such a way that at each point (x, y) on the curve the tangent line has slope $-\sin x$. Find an equation for the curve, given that it passes through the point (0, 2).

12. A ball is hit directly upward with an initial velocity of 49 m/s and is struck at a point that is 1 m above the ground. Assuming that the free-fall model applies, how high will the ball travel?

13. Suppose that a particle moves with acceleration $a(t) = \frac{1}{\sqrt{5t+1}}\ m/s^2$ along x axis and has velocity $v_0 = 2$ m/s at time $t = 0$. Find the displacement and the distance traveled by the particle during the time interval $0 \le t \le 3$.

14. Find the average value of the function $f(x) = \frac{\cos(\pi/x)}{x^2}$ over the Interval [1, 3].

15. A particle moves through 3-space in such a way that its velocity is $v(\vec{t}) = \hat{i} + t\hat{j} + t^2\hat{k}$. Find the coordinates of the particle at time $t = 1$ given that the particle at the point (–1, 2, 4) at time $t = 0$.

16. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}, y = 2$ and $x = 0$ is revolved about the y – axis.

17. Use polar coordinates to evaluate $\iint_R e^{-(x^2+y^2)}\, dA$, where R is the region enclosed by the circle $x^2 + y^2 = 1$.

18. Find the arc length parametrization of the circular helix $\vec{r} = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$ that has reference point $r(\vec{0}) = (1, 0, 0)$ and the same orientation as the given helix.

19. Using vector equation of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, find the curvature of the ellipse at the end points of the major and minor axes. Deduce the curvature of the circle.

20. Suppose that the speed of a particle at time t be $\|v(\vec{t})\| = \sqrt{3t^2 + 4}$. Find the scalar tangential component of acceleration at time $t = 2$.

21. Find the directional derivative of $f(x, y) = e^{xy}$ at (–2, 0) in the direction of the unit vector that makes an angle of $\frac{\pi}{3}$ with the positive x – axis.

22. Find the work done by the force field $\vec{F}(x, y) = xy\hat{i} + x^2\hat{j}$ on a particle that moves along the curve $x = y^2$ from (0,0) to (1,1).

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry **4** marks each.

23. A car that has stopped at a toll booth leaves the booth with a constant acceleration of 2 ft/s^2. At the time the car leaves the booth it is 5000 ft behind a truck traveling with a constant velocity of 50 ft/s. How long will it take for the car to catch the truck and how far will the car be from the toll booth at that time.

24. Sketch the region enclosed by the curves $x^2 = y$, $x = y - 2$ and find its area.

25. Prove that (i) $\nabla\|\vec{r}\| = \frac{\vec{r}}{\|\vec{r}\|}$ and (ii) $\nabla\frac{1}{\|\vec{r}\|} = -\frac{\vec{r}}{\|\vec{r}\|^3}$.

26. Use double integral to find the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0, 1] \times [0, 2]$.

27. Evaluate $\int_0^2 \int_{y/2}^1 \cos(x^2)\,dx\,dy$ by changing the order of integration.

28. Use cylindrical coordinates to find the volume of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$.

29. A shell is to be fired from ground level at an elevation angle of 30°. What should be the muzzle speed be in order for the maximum height of the shell to be 2500 ft.

30. Show that the divergence of the inverse square field $\vec{F}(x, y, z) = \frac{c}{(x^2 + y^2 + z^2)^{3/2}}(x\hat{i} + y\hat{j} + z\hat{k})$ is zero.

31. Evaluate the line integral $\int_C (xy + z^3)ds$ from (1,0,0) to (–1,0, π) along the helix C that is represented by the parametric equation $x = \cos t$, $y = \sin t$, $z = t$ $(0 \le t \le \pi)$.

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) Find the area of the region enclosed by the curves $y = 2 + |x - 1|$ and $y = -\frac{1}{5}x + 7$. (7)

(b) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x – axis. (8)

33. (a) Evaluate $\iiint_G y\,dV$ where G is the solid enclosed by the plane $z = y$, the xy – plane and the parabolic cylinder $y = 1 - x^2$ (8)

(b) Find the total arc length of the cardioid $r = 1 + \cos\theta$. (7)

34. (a) Show that the force field $\vec{F}(x,y) = 2xy^3\hat{i} + 3x^2y^2\hat{j}$ is conservative in any open connected region containing the points $P(-3, 0)$ and $Q(4, 1)$ and then find the work done by the force field on a particle moving along any smooth curve in the region from P to Q. (8)

(b) Evaluate the surface integral $\iint_{G} \sigma xz dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant (7)

35. Use the Divergence theorem to find the outward flux of the vector field $\vec{F}(x,y,z) = x^3\hat{i} + y\hat{j} + z^3\hat{k}$ across the surface of the region that is enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$.
