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**Reg. No. :** ..............................

**Name :** ..................................

**Sixth Semester B.Sc. Degree Examination, March 2021**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course XII**

**MM 1644 : LINEAR ALGEBRA**

**(2018 Admission Regular)**

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer all the first **ten** questions. Each carries **1** mark :

1. Find a point with $z=2$ on the intersection line of the planes $x+y+3z=6$ and $x-y+z=4$.

2. Find the inverses of $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$.

3. Give examples of $A$ and $B$ such that $A+B$ is not invertible although $A$ and $B$ are invertible.

4. Define Null Space of a matrix $A$.

5. Show that the vectors $(1, 0)$ and $(0, 1)$ are linearly independent.

P.T.O.

6. True or false? "The determinant of $S^{-1}AS$ equals the determinant of $A$".

7. Write Cramer's Rule

8. Find the sum of eigen values of $\begin{bmatrix} 3 & 2 & 5 & 7 \\ 7 & 1 & 4 & 9 \\ 6 & 9 & 11 & 23 \\ 1 & 3 & 77 & -5 \end{bmatrix}$.

9. Is the matrix $\begin{bmatrix} 3 & 3 & 15 \\ 0 & 4 & 8 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable and why?

10. Find the inner product of $\begin{bmatrix} 1+i \\ 2-3i \end{bmatrix}$ and $\begin{bmatrix} 1-i \\ 2+3i \end{bmatrix}$.

**(10 × 1 = 10 Marks)**

SECTION – II

Answer **any eight** questions among the questions 11 to 26. They carry **2** marks each.

11. Find the equation of a line that meets $x+4y=7$ at $x=3$, $y=1$.

12. The matrix $A(\theta)$ that rotates the $x-y$ plane by an angle $\theta$ is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Show that $A(\theta_1)A(\theta_1)=A(\theta_1+\theta_2)$.

13. For which three numbers $c$ is this matrix not invertible, and why not?

$$\begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}.$$

14. Check whether $S=\{[a\,b\,c]\in R^3 | a=b+1\}$ is a subspace of $R^3$.

15. Show that the set of all solutions of the set of linear equations $Ax=b, b\neq 0$, under standard matrix addition and scalar multiplication is not a vector space.

16. Decide the dependence or independence of the vectors $(1,3,2)$, $(2,1,3)$, and $(3,2,1)$.

17. Determine whether the transformation $T$ is linear if $T:R^2 \mapsto R^1$ is defined by $T[a\ b]=ab$ for all real numbers $a$ and $b$.

18. Let $S:M_{n\times n}\mapsto R$ map an $n\times n$ matrix into the sum of its diagonal elements. Such a transformation is known as the trace. Is it linear?

19. The corners of a triangle are $(2,1)$, $(3,4)$, and $(0,5)$. What is the area?

20. Solve $3u+2v=7$, $4u+3v=11$ by Cramers rule.

21. Suppose $(x, y, z)$ is a linear combination of $(2,3,1)$ and $(1,2,3)$. What determinant is zero? What equation does this give for the plane of all combinations?

22. Suppose that $\lambda$ is an eigenvalue of A, and $x$ is its eigenvector: $Ax=\lambda x$. Show that this same $x$ is an eigenvector of $B=A-7I$, and find the eigenvalue.

23. If a 3 by 3 upper triangular matrix has diagonal entries 1, 2, 7, how do you know it can be diagonalized? What is A?

24. If B has eigenvalues 1, 2, 3. $C$ has eigen values 4, 5, 6, and $D$ has eigenvalues 7, 8, 9, what are the eigen values of the 6 by 6 matrix

$$A=\begin{bmatrix} B & D \\ 0 & C \end{bmatrix}.$$

25. Prove that " If $A=A^H$, every eigenvalue is real".

26. If $A+iB$ is a Hermitian matrix ($A$ and $B$ are real), show that $\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$ is symmetric.

**(8 × 2 = 16 Marks)**

## SECTION – III

Answer **any six** questions among the questions 27 to 38. They carry **4** marks each.

27. Solve by Gauss Elimination Method

$$x + y + z = 3$$
$$x + 2y + 3z = 0$$
$$x + 3y + 2x = 3$$

28. What three elimination matrices $E_{21}, E_{31}$, and $E_{32}$ put $A$ into upper-triangular form $E_{21}.E_{31}E_{32} = U$ Using these, compute the matrix $L$ (and $U$) to factor $A = LU$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{bmatrix}.$$

29. Determine whether the set of column matrices in $R^3 \left\{ \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 8 \\ 16 \\ -3 \end{bmatrix} \right\}$ linearly independent.

30. By applying row operations to produce an upper triangular $U$, compute

$$\det \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix}.$$

31. Find a basis for the row space of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & -1 & 1 \\ 3 & 2 & 5 \\ 5 & 15 & 20 \end{bmatrix}.$

32. A linear transformation $T: R^2 \to R^2$ has the property that

$$T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \text{ and } T\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Determine Tv for any vector $v \in R^2$.

33. Use row operations to verify that the 3 by 3 "Vandermonde determinant" is

$$\det\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (b-a)(c-b)(c-a).$$

34. Evaluate this determinant by cofactors of row 1: $\begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix}$.

35. Find $x$, $y$, and $z$ by Cramers Rule in equation

$$x + 4y - z = 1$$
$$x + y + z = 0$$
$$2x + 3z = 0$$

36. Diagonalize the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$.

37. Prove that "Diagonalizable matrices share the same eigenvector matrix $S$ if and only if $AB = BA$".

38. Write the matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ in the form $U^{-1}AU = T$.

**(6 × 4 = 24 Marks)**

SECTION – IV

Answer **any two** questions among the questions 39 to 44. They carry **15 marks.**

39. (a) Which number $q$ makes this system singular and which right-hand side t gives it infinitely many solutions? Find the solution that has $z=1$.

$$x+4y-2z=1$$
$$x+7y-6z=6$$
$$3y+qz=t$$

(b) For the system

$$u+v+w=2$$
$$u+3v+3w=0$$
$$u+3v+5w=2$$

what is the triangular system after forward elimination, and what is the solution?

40. (a) Using the Gauss-Jordan Method to Find $A^{-1}$

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}.$$

(b) Show that the set of all even functions defined on $R$, that is $f(x)=f(-x)$ is a vector space.

41. (a) Determine whether the transformation $R$ is linear, if $R$ is defined by

$$R\begin{bmatrix} a \\ b \end{bmatrix}=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}=\begin{bmatrix} a\cos\theta & -b\sin\theta \\ a\sin\theta & +b\cos\theta \end{bmatrix}$$

where $a$ and $b$ denote arbitrary real numbers and $\theta$ is a constant.

(b) Find a basis for each of these subspaces of 3 by 3 matrices:

(i) All diagonal matrices.

(ii) All symmetric matrices.

42. (a) By applying row operations to produce an upper triangular $U$, compute

$$\det\begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix}.$$

(b) Suppose the permutation $P$ takes $(1, 2, 3, 4, 5)$ to $(5, 4, 1, 2, 3)$

(i) What does $P^2$ do to $(1, 2, 3, 4, 5)$?

(ii) What does $P^{-1}$ do to $(1, 2, 3, 4, 5)$?

43. Let $A = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

(a) Find all eigen values of A.

(b) Find a maximum set of linearly independent eigen vectors of $A$

(c) Is A diagonalizable? If yes, find S such that $A = S \wedge S^{-1}$.

44. (a) $B$ is similar to $A$ and $C$ is similar to $B$, show that $C$ is similar to $A$

(b) Explain why $A$ is never similar to $A + I$

(c) Show (if $B$ is invertible) that $BA$ is similar to $AB$.

**(2 × 15 = 30 Marks)**

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