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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, February 2021

First Degree Programme under CBCSS

Mathematics

Core Course V

MM 1541 : REAL ANALYSIS — I

(2015 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

All the first 10 questions are compulsory. They carry **1** mark each.

1. If u and $b \neq 0$ are real numbers such that $u \cdot b = b$, then $u =$ ————.

2. Give a lower bound of $\{x \in \mathbb{R} : -3 < x \leq 1\}$.

3. Give an example of an unbounded interval.

4. State the infimum property of $\mathbb{R}$.

5. If X and Y are the sequences $X = (5n)$, and $Y = \left(\frac{1}{3n}\right)$, then the product sequence $\left(\frac{X}{Y}\right)$ is ————.

6. Give an example of a divergent sequence.

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7. $\lim \dfrac{2n + 1}{n + 5}$ = ____________.

8. The set of cluster points of the set $A = [0, 1]$ is ____________.

9. $\lim\limits_{x \to c} x$ = ____________.

10. $\lim\limits_{x \to -3} \dfrac{1}{(x + 3)^2}$ = ____________.

Answer **any eight** questions from among the questions 11 to 22, these questions carry **2** marks each.

11. If $a \in \mathbb{R}$, then show that $(-a) \cdot (-a) = a^2$.

12. Prove that $1 > 0$.

13. Show that if a nonempty set S has finite number of elements, then $\sup S$ exists and belongs to the set S.

14. Describe Dedekind's form of completeness property.

15. Show that $\lim\left(\dfrac{1}{n^2}\right) = 0$.

16. Prove that a Cauchy sequence of real numbers is bounded.

17. Prove that every contractive sequence is a Cauchy sequence.

18. Prove that $\sum\limits_{n=0}^{\infty} ar^n$ converges if $|r| < 1$.

19. Show that the series $\dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + ...$ converges.

20. Prove that $\lim\limits_{x \to 2} \dfrac{x^3 - 4}{x^2 + 1} = \dfrac{4}{5}$.

21. Prove that $\lim_{x \to 0} \sin(x)$ does not exist.

22. If p and q are polynomial function on $\mathbb{R}$ and if $q(c) \neq 0$, then prove that $\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{p(c)}{q(c)}$.

Answer **any six** questions from the questions 23 to 31. These questions carry **4** marks each.

23. If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every positive ε, then prove that $a = 0$.

24. If a and b are real numbers, prove that $||a| - |b|| \leq |a - b|$.

25. If $x \in \mathbb{R}$ then show that there exists $n_x \in \mathbb{N}$ such that $x < n_x$.

26. If $I_n = [a_n, b_n], n \in \mathbb{N}$, is a nested sequence of closed and bounded intervals, then prove that there exists a number $\xi \in \mathbb{R}$ such that $\xi \in I_n$ for all $n \in \mathbb{N}$.

27. Prove that a sequence of real numbers can have at most one limit.

28. Prove that a bounded sequence of real numbers has a convergent subsequence.

29. If $X = (x_n)$ is a sequence of real numbers, then prove that there is a subsequence of X that is monotone.

30. Prove that a montone sequence of real numbers is properly divergent if and only if it is unbounded.

31. If $f : A \to \mathbb{R}$ and if c is a cluster point of A, then prove that f can have at most one limit at c.

Answer **any two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) Determine the set of all real numbers x that satisfy $|x - 1| < 1$.

(b) Let A and B be bounded nonempty subsets of the set of real numbers, and let $A + B = \{a + b : a \in A, b \in B\}$, prove that $\sup(A + B) = \sup A + \sup B$.

33. (a) Prove that a monotone sequence of real numbers is convergent if and only if it is bounded. Also, prove that if $Y = (y_n)$ is a bounded decreasing sequence, then $\lim(y_n) = \inf\{y_n : n \in \mathbb{N}\}$.

(b) Examine the convergence of the sequence (e_n), where $e_n = \left(1 + \frac{1}{n}\right)^n$ for $n \in \mathbb{N}$.

34. (a) Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.

(b) Discuss the convergence of the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ if converges, find the sum.

35. (a) Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ are two functions, and let $c \in \mathbb{R}$ be a cluster point of A. If $\lim_{x \to c} f = L$ and $\lim_{x \to c} g = M$ (L and M real numbers), then prove that $\lim_{x \to c} (f + g) = L + M$

(b) Show that $\lim_{x \to 0} \sin x = 0$.

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