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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS — II

(2018 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Evaluate $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$.

2. State true or false : Every continuous function in a closed interval is bounded.

3. Determine the points of discontinuity of the greatest integer function.

4. State Rolle's theorem.

5. Define a uniformly continuous function.

6. Define a differentiable function at a point.

P.T.O.

7. Give an example of a real valued function which is discontinuous at every point of R.

8. Define upper integral of a function f.

9. When do you say that a bounded real function f is integrable on $[a, b]$?

10. State true or false : If $|f|$ is integrable on $[a, b]$ then f is also integrable on $[a, b]$.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions (11 – 26). Each question carries **2** marks.

11. Evaluate the limit : $\lim_{x \to 2} \frac{|x-2|}{x-2}$.

12. Let f and g be real valued functions then prove that $\lim_{x \to c} \{f(x)g(x)\} = \lim_{x \to c} f(x) . \lim_{x \to c} g(x)$.

13. Prove that the Dirichlet's function f defined on R by $f(x) = \begin{cases} 1 \text{ when } x \text{ is irrational} \\ -1 \text{ when } x \text{ is rational} \end{cases}$ is discontinuous at every point.

14. If f, g be two functions continuous at a point c, then the function $f+g$ is also continuous at c.

15. Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

16. Prove that $\{f(x_n)\}$ is a Cauchy sequence for every Cauchy sequence $\{x_n\}$ in R where f is a uniformly continuous function.

17. If f is differentiable in (a, b) and $f'(x) \leq 0$ for all $x \in (a, b)$, show that f is monotonically decreasing.

18. Show by an example that a bounded function in $[a, b]$ need not be continuous in $[a, b]$.

19. Let f be defined on $[a, b]$. If f is differentiable at a point $c \in [a,b]$, then f is continuous at c.

20. Suppose f and g are defined on $[a, b]$ and are differential at a point $x \in [a,b]$. Prove that fg is differentiable.

21. Give an example to show that continuous function need not be differentiable.

22. Check whether the following function is integrable over $[0, 1]$: $f(x)=1$ if $x \in [0,1]$ and x is rational and $f(x)=0$ if $x \in [0, 1]$ and x is irrational.

23. Show that $\int_a^b f dx \leq \int_a^b f\, dx$.

24. If P and Q are two partitions of $[a, b]$ and $P \subseteq Q$ then for a bounded function f, prove that $U(Q,f) - L(Q,f) \leq U(P, f) - L(P, f)$.

25. Show that if f and g are bounded and integrable on $[a, b]$, such that $f \geq g$, then $\int_a^b f\, dx \geq \int_a^b g\, dx$.

26. If f is bounded and integrable in $[a, b]$. Prove that there exists a number μ lying between a and b such that $\int_a^b f(x)dx - \mu(b-a)$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions (27 - 38). Each question carries **4** marks.

27. Show that the limit $\lim_{x \to 0}\left(\sin\frac{1}{x}\right)$ does not exist.

28. State and prove extreme value theorem.

29. Prove that a function which is continuous on a closed interval is uniformly continuous on that interval.

30. Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

31. Prove that if f is differentiable on an open interval in (a, b) and f attains a minimum value at some point c in (a, b) then $f'(c) = 0$.

32. State and prove chain rule of differentiation.

33. State and prove Darboux's theorem.

34. Compute $\int_{1}^{1} f\,dx$ where $f(x) = |x|$.

35. State and prove Mean value theorem.

36. Prove that if f is monotonic in $[a, b]$, then f is integrable in $[a, b]$.

37. If f and g are integrable in $[a, b]$ then show that fg is also integrable in $[a, b]$.

38. Prove that a continuous function in a closed interval is integrable in that interval.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions (39 - 44). Each question carries **15** marks.

39. Define Lipschitz functions. Show that every Lipschitz function is uniformly continuous. Is the converse true? Justify.

40. State and prove Intermediate value theorem. Is the converse true? Justify.

41. State and prove chain rule for differentiation.

42. Suppose f and g are real and differentiable in (a,b) and that $f'(x) \neq 0$ for all $x \in (a, b)$. If $\lim_{x \to a} g(x) = +\infty$ then show that $\lim_{x \to a} \frac{g'(x)}{f'(x)} - L$ implies $\lim_{x \to a} \frac{g(x)}{f(x)} = L$.

43. A bounded function f is integrable on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f) - L(P, f) < \varepsilon$.

44. State and prove fundamental theorem of integral calculus.

(2 × 15 = 30 Marks)