

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme under CBCSS

Complementary Course for Mathematics

ST 1331.1 – STATISTICAL DISTRIBUTIONS

(2018 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. If the standard deviation of a Poisson random variable  $X$  is 3, write the probability mass function of  $X$ .
2. Moment generating function of a random variable  $Y$  is  $(0.65 + 0.35e^t)^5$ . Identify the statistical distribution and its parameters.
3. Write the mean and variance of geometric distribution.
4. Write the mode for the Poisson distribution with mean 7.5.
5. Let  $X$  follows discrete uniform with parameter  $n$ . Compute the coefficient of variation of  $X$ .
6. What is odd order moment about mean of normal distribution?
7. Define statistic.
8. Define  $t$  statistic.

9. Let  $Y$  be a random variable and  $Y$  follows exponential distribution with mean 3. Compute  $P(Y=3)$ .
10. Write the variance of random variable follows Chi square distribution with 10 degrees of freedom.

**(10 × 1 = 10 Marks)**

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define Bernoulli distribution. What is its mean?
12. If  $X$  follows Binomial  $(n,p)$ , derive the distribution of  $n-X$ .
13. Define hyper geometric distribution.
14. Derive the MGF of a discrete uniform random variable.
15. Let  $X$  be a continuous uniform random variable with mean 1 and variance  $4/3$ . Find  $P(X<0)$ .
16. State the additive property of gamma distribution.
17. Write the relationship between Beta I and Beta II random variables.
18. Let  $X$  be standard Normal random variable, compute  $P(1 < X < 2)$ .
19. Define convergence in probability.
20. State central limit theorem for i.i.d random variables.
21. Justify the statement "every statistic is a random variable".
22. Write probability density functions of  $t$  and  $F$  distributions.

**(8 × 2 = 16 Marks)**

### SECTION – C

Answer **any six** questions. Each question carries **4** marks.

23. If  $X \sim \text{Binomial}(6, p)$  and  $P(X = 4) = P(X = 2)$ , find the value of  $p$ .
24. Let  $X_1$  and  $X_2$  be independent and identically distributed geometric random variables. Show that the conditional distribution of  $X_1$  given  $X_1 + X_2$  is uniform.
25. For a Normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. What is the mean and variance of the distribution?
26. Derive the mean and variance of Beta I distribution.
27. State and prove lack of memory property of exponential distribution.
28. Derive Bernoulli's law of large numbers.
29. Derive the moment generating function of Chi square distribution and hence derive its mean and variance.
30. Let  $X_n$  assumes the values  $\frac{1}{\sqrt{n}}$  and  $-\frac{1}{\sqrt{n}}$  with probabilities  $\frac{2}{3}$  and  $\frac{1}{3}$  respectively. Check whether the weak law of large numbers holds good for the sequence  $\{X_n\}$  of independent random variables.
31. Derive the relationships between Chi square,  $t$  and  $F$  distributions.

**(6 × 4 = 24 Marks)**

### SECTION – D

Answer **any two** questions. Each question carries **15** marks.

32. (a) Derive the recurrence relation for central moments of Poisson distribution.  
(b) Prove that under certain conditions Binomial distribution tends to Poisson distribution.
33. (a) Define Normal distribution.  
(b) Derive the mean, median and mode of Normal distribution.

34. (a) Derive Chebyshev's inequality.
- (b) Suppose that the lifetime of an electronic device follows exponential distribution with mean 1. Determine the upper bound of  $P(|X - 1| \geq 2)$  using Chebyshev's inequality.
35. (a) Let  $X_1$  and  $X_2$  be two independent random variables follow Chi square distribution with 1 degrees of freedom. Determine the value of  $k$  if  $P(X_1 + X_2 > k) = 0.5$ .
- (b) Establish the sampling distribution of the sample variance of random sample drawn from Normal distribution.

**(2 × 15 = 30 Marks)**