

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme under CBCSS

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS III – VECTOR ANALYSIS AND THEORY OF  
EQUATIONS

(2014-2017 Admissions)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. What is your observation about imaginary roots of a polynomial equation?
2. State Des-Carte's rule of signs of a polynomial equation.
3. Form a rational cubic equation whose roots include 3 and  $\sqrt{2}$ .
4. Give an example for transcendental equation.
5. Define the continuity of a vector function.
6. Write down the condition that  $Mdx + Ndy + Pdz$  is exact.
7. Define  $\text{div } \phi$ .
8. Define the line integral of  $F$  along  $C$ .

9. If  $\vec{r} = xi + yj - zk$ , evaluate  $\text{curl } \vec{r}$ .

10. State Gauss divergence theorem.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Solve the equation  $2x^3 - 9x^2 - 27x + 54 = 0$ , given that its roots are in GP.
12. Solve the equation  $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ , given that two of its roots are equal in magnitude and opposite in sign.
13. Solve  $2x^3 - 7x^2 + 36 = 0$  given that the difference between two of the roots is 5.
14. Find the condition that the roots of  $ax^3 + 3bx^2 + 3cx + d = 0$  may be in GP.
15. Find the unit tangent vector to the curve  $x = t^2 + 1$ ,  $y = 4t - 3$ ,  $z = 2t^2 - 6t$  at the point  $t = 2$ .
16. The position vector of a particle in space at time  $t$  is  $r(t) = e^{-t}i + 2\cos 3tj + 2\sin 3tk$ . Find the velocity and acceleration vectors.
17. If  $F = 6x^2zi + 2x^2yj - yz^2k$ , find  $\text{div } F$ .
18. Find the work done by  $F = xyi + yj - yzk$ , over the curve  $r(t) = ti + t^2j + tk$ ,  $0 \leq t \leq 1$ , in the direction of increasing  $t$ .
19. Integrate  $f(x, y, z) = x^2 + z^2$  over the circle  $r(t) = a\cos tj + a\sin tk$   $0 \leq t \leq 2\pi$ .
20. Show that curvature of a circle of radius  $a$  is  $\frac{1}{a}$ .
21. Find the length of the curve  $r(t) = \sqrt{2}ti + \sqrt{2}tj + (1-t)k$  from  $(0, 0, 1)$  to  $(\sqrt{2}, \sqrt{2}, 0)$ .
22. Establish the relation  $\nabla \cdot (\nabla \times \vec{F}) = 0$ .

(8 × 2 = 16 Marks)

## PART – C

Answer any **six** questions. **Each** question carries **4** marks.

23. If  $\alpha, \beta, \gamma, \delta$  are the roots of the equation  $ax^4 + bx^3 + cx^2 + dx + e = 0$  such that  $\alpha + \beta = \gamma + \delta$ , show that  $4abc - b^3 - 8a^2d = 0$ .
24. Solve the equation  $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$  whose roots are in AP.
25. Find the root of the equation  $x^3 - 9x - 1 = 0$  correct to three decimal places using bisection method.
26. Describe the Newton-Raphson method of finding the solution of a general function  $f(x) = 0$ .
27. Establish the relation  $\nabla \cdot (\nabla \times \vec{F}) = 0$ .
28. Using Green's theorem calculate the integral  $\oint_C y^2 dx + x^2 dy$  where  $C$  is the boundary of the triangle bounded by  $x = 0$ ,  $x + y = 1$ ,  $y = 0$  in the counterclockwise direction.
29. Find the flux of  $F = (x - y)\mathbf{i} + x\mathbf{j}$  across the circle  $x^2 + y^2 = 1$  in the  $xy$ -plane.
30. Find the potential function for the field  $(e^x \cos y + yz)\mathbf{i} + (xz - e^x \sin y)\mathbf{j} + (xy + z)\mathbf{k}$ .
31. Show that gradient field describing a motion is irrotational.

**(6 × 4 = 24 Marks)**

## PART – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Solve the equation  $4x^4 - 85x^3 + 357x^2 - 340x + 64 = 0$ , given that roots are in geometric progression.
- (b) Solve the equation  $15x^3 - 23x^2 + 9x - 1 = 0$  whose roots are in HP.

33. Use Newton-Raphson method to obtain a root, correct to three decimal places of  $x - \cos x = 0$ .
34. If  $A = 2xyi + yz^2j + xzk$  and  $S$  is a rectangular parallelepiped bounded by  $x = 0, y = 0, z = 0, x = 2, y = 1, z = 3$  verify divergence theorem.
35. Verify Stoke's theorem, when  $F = (2x - y)i - yz^2j - y^2zk$ ,  $S$  is the upper half surface of the unit sphere  $x^2 + y^2 + z^2 = 1$  and  $C$  is its boundary.

**(2 × 15 = 30 Marks)**

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