

C 2217

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Name.....

Reg. No.....

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2021

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all the twelve questions.

Each question carries 1 mark.

1. What do you mean by a non-linear differential equation ?
2. What are the steps for finding general solution of a non-homogeneous equation $y'' + ay' + by = r(x)$.
3. Find Wronskian of $y_1(x) = e^{-2x}$ and $y_2(x) = e^{-3x}$.
4. What is $L[1]$?
5. Define periodic function.
6. What is unit step function ?
7. State Convolution theorem.
8. Define and give an example of an even function.
9. Give one dimensional wave equation.
10. Write the formula for Runge Kutta method.
11. Give formula for Euler method.
12. Give a formula for an error for Simpson's rule.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any nine questions.
Each question carries 2 marks.

13. Find the particular integral for $y'' - 4y' + 3y = 10e^{-2x}$.
14. Solve $(D^2 - 2D + 3)y = x^3 + \sin x$.
15. Find $W[e^{\lambda_1 x}, e^{\lambda_2 x}]$.
16. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at} F(t)$.
17. Show that the Laplace transform is a linear operation.
18. Find $L[t^2 \cos t]$.
19. Using convolution property, find $L^{-1}\left[\frac{1}{s^2(s-a)}\right]$.
20. Find the Fourier series of $f(x) = x^2$, when $-1 < x < 1$ with period 2.
21. Show that $u = \cos 4t \sin 2x$ is a solution of the wave equation.
22. Apply Picard's iteration upto 3 steps to solve $y' = 1 + y^2$ and $y(0) = 1$.
23. Compute $\int_0^1 x^2 dx$ by the rectangular rule with $h = 0.5$.
24. Solve $\int_1^2 \frac{1}{x} dx$ by Trapezoidal rule with $n = 4$ and compare the estimate with the exact value of the integral.

(9 × 2 = 18 marks)

Part C

Answer any six questions.
Each question carries 5 marks.

25. Solve $x^2 y'' + 7xy' + 13y = 0$.
26. Solve the non-homogeneous equation $y'' - 4y' + 3y = 10e^{-2x}$.
27. Obtain the Fourier cosine series representation of $f(x) = e^x, x \in [0, \pi]$.
28. Find the inverse transform of $\frac{s^3 - 4s^2 + 4}{s^2(s^2 - 3s + 2)}$.
29. Solve $u_x + u_y = 2(x + y)u$.
30. Express the function $f(x) = x^2$, when $-1 < x < 1$ as a Fourier series with period 2.
31. Solve the integral equation $y = 1 - \int_0^t (t - \tau) y(\tau) d\tau$.
32. Find an approximate value of $\log_e 5$ by calculating $\int_0^5 \frac{dx}{4x + 5}$ by Simpson's rule of integration.
33. Solve by Picard's method $y' - xy = 1$, given $y = 0$ when $x = 2$. Also find $y(2.05)$ correct to four places of decimal.

(6 × 5 = 30 marks)

Part D

Answer any two questions.
Each question carries 10 marks.

34. (a) Solve $x^2 y'' - 4xy' + 6y = 21x^{-4}$.
- (b) Solve $(D^2 - 2D + 1)y = 3x^{3/2}e^x$.

Turn over

35. Find the solution of the wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

corresponding to the triangular initial deflection

$$f(x) = \begin{cases} \frac{2k}{l}x, & \text{when } 0 < x < \frac{l}{2} \\ \frac{2k}{l}(l-x), & \text{when } \frac{l}{2} < x < l \end{cases}$$

and the initial velocity zero.

36. Find the Fourier series of $f(x) = \begin{cases} 2, & -2 \leq x < 0 \\ x, & 0 \leq x < 2 \end{cases}$ in $(-2, 2)$.

(2 × 10 = 20 marks)