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Name.....

Reg. No.....

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2020

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019 Admissions)

Time : Two Hours

Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. A train has position $x = 3t^2 + 2 - \sqrt{t}$ at time t. Find the velocity of the train at $t = 2$.

2. Find $\lim_{x \to 2} \frac{-3x}{x^2 - 4x + 4}$.

3. Find the slope of the line tangent to the graph of $f(x) = x^8 + 2x^2 + 1$ at $(1, 4)$.

4. Suppose that $f(t) = \frac{1}{4}t^2 - t + 2$ denotes the position of a bus at time t. Find and plot the speed as a function of time.

5. Find $\frac{d^2}{dr^2}\left(8r^2 + 2r + 10\right)$.

6. If $x^2 + y^2 = 3$, compute $\frac{dy}{dx}$ when $x = 0$ and $y = \sqrt{3}$.

7. On what interval is $f(x) = x^3 - 2x + 6$ increasing or decreasing ?

Turn over

18. An object on the x-axis has velocity $v = 2t - t^2$ at time t. If it starts out at $x = -1$ at time $t = 0$, where is at time $t = 3$? How far has it traveled?

19. Find average value of $f(x) = x^2 \sin x^3$ on $[0, \pi]$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Using product rule, differentiate $\left(x^2 + 2x - 1\right)\left(x^3 - 4x^2\right)$. Check your answer by multiplying out first.

(b) Find the dimensions of a rectangular box of minimum cost if the manufacturing costs are 10 cents per square meter on the bottom, 5 cents per square metre on the sides, and 7 cents per square metre on the top. The volume is to be 2 cubic meters and height is to be 1 metre.

21. (a) The curves $y = x^2$ and $x = 1 + \frac{1}{2}y^2$ divide the xy plane into five regions, only one of which is bounded. Sketch and find the area of this bounded region.

(b) The region between the graph of x^2 on $[0, 1]$ is revolved about the x-axis. Sketch the resulting solid and find its volume.

(1 × 11 = 11 marks)

$(x^2 + 2x - 1)(x^3 - 4x^2)$

$x^5 + 4x^4 + 2x^4 - 8x^2 - x^3 - 4x^2$

$x^5 - 2x^4 - x^3 - 12x^2$

8. Use the second derivative test to analyze the critical points of the function $f(x) = x^3 - 6x^2 + 10$.

9. Discuss the concavity of $f(x) = 4x^3$ at the points $x = -1$ and $x = 1$.

10. Find $\int_2^6 (x^2 + 1)\,dx$.

11. Find the area between the graph of $y = x^2$ and $y = x^3$ for x between 0 and 1.

12. Find the average value of $f(x) = x^2$ on $[0, 2]$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. (a) Find $\frac{d}{dx}\left(\frac{\sqrt{x}}{1 + 3x^2}\right)$.

(b) Calculate approximate value for $\sqrt{9.02}$ using linear approximation around $x_0 = 9$.

14. Find the equation of the tangent line to the curve $2x^6 + y^4 = 9xy$ at the point $(1, 2)$.

15. Find the slope of the parametric curve given by $x = \left(1 + t^3\right)^4 + t^2$, $y = t^5 + t^2 + 2$ at $t = 1$.

16. State mean value theorem. Verify mean value theorem for the function $f(x) = x^2 - x + 1$ on $[-1, 2]$.

17. Find $\lim_{x \to 0}\left(\frac{1}{x \sin x} - \frac{1}{x^2}\right)$.