C 81806 (Pages : 3) Name..................................

Reg. No..................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all twelve** *questions.*

Each question carries 1 mark.

1. Find a formula for *n*th term of the sequence 1, –1, 1, –1,.......
2. Define a non-decreasing sequence.
3. Write tanh x in terms of exponential function.
4. State Sandwich theorem for sequences.
5. Find the domain of the function $w = \sqrt{y - x^2}$.
6. Define contour line of a function $f(x, y)$.
7. Find $\lim_{(x,y)\to(0,1)} \frac{2 - xy + 3}{x^2 y + 5xy - y^3}$.
8. Define absolute convergence of a series.
9. $\frac{d}{dx}\sinh 2x =$ ————.
10. Define level curve of a function.
11. Find $\lim_{n\to\infty} n^{1/n}$.
12. $\int_0^1 \sinh x dx =$ ————.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*

Each question carries 2 marks.

13. Is the area under the curve $y = 1/\sqrt{x}$ from $x = 0$ to $x = 1$ finite ? If so, what is it ?
14. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$.

Turn over

15. Show that $\lim_{n\to\infty} \frac{1}{n} = 0$.

16. Determine whether the sequence $a_n = \frac{2^n 3^n}{n!}$ is non-decreasing and bounded from above.

17. Show that $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y) \neq (0,0); \\ 0, & (x,y) = (0,0); \end{cases}$ is continuous at every point except the origin.

18. Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4,–5) if $f(x,y) = x^2 + 3xy + y - 1$.

19. Find the volume of the solid generated by revolving the region between y-axis and the curve $x = 2/y, 1 \leq y \leq 4$ about the y-axis.

20. Find $\int_0^{ln2} 4e^x \sin x dx$.

21. Graph the sets of points whose polar co-ordinates satisfies the conditions $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$.

22. Replace the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$ equivalent cartesian equation.

23. Find the spherical co-ordinates equation for the cone $z = \sqrt{x^2 + y^2}$.

24. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$. What is the derivative's value at $t = 0$?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

26. Show that $(-1)^n \frac{n-1}{n}$ diverges.

27. Find the area of the region in the plane enclosed by the cardioid $r = 2(1+\cos\theta)$.

28. Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

29. Find the linearization of $f(x,y,z) = x^2 - xy + 3\sin z$ at the point (x_0, y_0, z_0).

30. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$. What is the derivative's value at $t = 0$?

31. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

32. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 z}{\partial z^2} = 0$ if $f(x,y,z) = \left(x^2 + y^2 + z^2\right)^{-1/2}$.

33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)\left(x^2+1\right)} dx$.

35. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

36. Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \le x \le 4$ about the x-axis.

(2 × 10 = 20 marks)